

Theory and Phenomenology of Neutrino Oscillations and Masses

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Neutrino Unbound: <http://www.nu.to.infn.it>

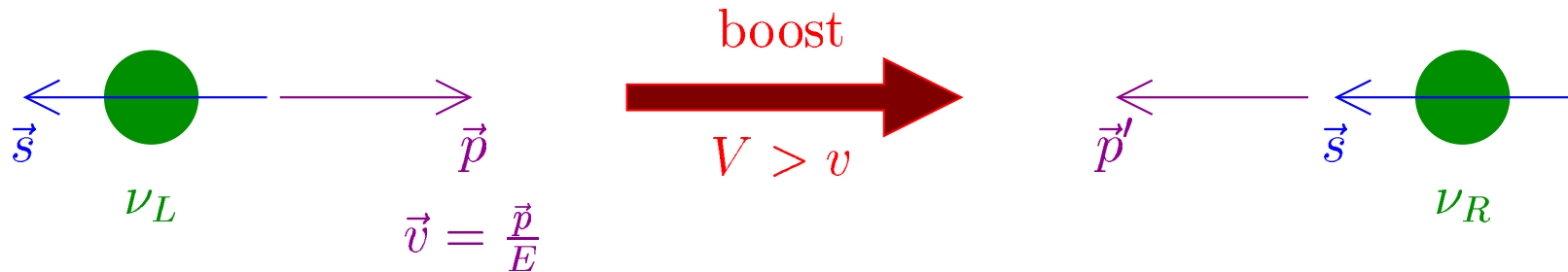
Tau06, Pisa, 21 Settembre 2006

Introduction to Neutrino Masses, Mixing and Oscillations

Solar $\nu_e \rightarrow \nu_\mu, \nu_\tau$ + Atmospheric $\nu_\mu \rightarrow \nu_\tau \implies$ 3- ν Mixing

Absolute Scale of Neutrino Masses

Neutrino Mass



Standard Model: $\nu_L, \nu_R^c \implies$ no Dirac mass term $\mathcal{L}^D \sim m^D (\overline{\nu}_L \nu_R + \overline{\nu}_R \nu_L)$
(no ν_R, ν_L^c)

Majorana Neutrino: $\nu \equiv \nu^c$

$\nu_R^c \equiv \nu_R \implies$ Majorana mass term $\mathcal{L}^M \sim m^M (\overline{\nu}_L \nu_R^c + \overline{\nu}_R^c \nu_L)$

Standard Model: Majorana mass term **not** allowed by $SU(2)_L \times U(1)_Y$
(no Higgs triplet)

Standard Model can be extended with ν_R ($e_L, e_R; u_L, u_R; d_L, d_R; \dots$)

$\nu_L + \nu_R \Rightarrow$ Dirac mass term $\mathcal{L}^D \sim m^D (\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) \Rightarrow m^D \lesssim 100 \text{ GeV}$

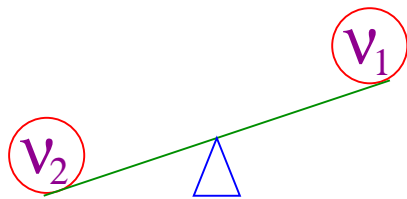
surprise: Majorana mass for ν_R is allowed! $\mathcal{L}_R^M \sim m_R^M (\bar{\nu}_L^c \nu_R + \bar{\nu}_R \nu_L^c)$

total neutrino mass term $\mathcal{L}^{D+M} \sim \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_L^c \end{pmatrix} \begin{pmatrix} 0 & m^D \\ m^D & m_R^M \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix}$

m_R^M can be arbitrarily large (not protected by SM symmetries)

$m_R^M \sim$ scale of new physics beyond Standard Model $\Rightarrow m_R^M \gg m^D$

diagonalization of $\begin{pmatrix} 0 & m^D \\ m^D & m_R^M \end{pmatrix} \Rightarrow m_1 \simeq \frac{(m^D)^2}{m_R^M}, \quad m_2 \simeq m_R^M$



natural explanation of
smallness of neutrino masses

see-saw mechanism

massive neutrinos are Majorana!

Standard Model:

Lepton numbers are conserved

	L_e	L_μ	L_τ		L_e	L_μ	L_τ
(ν_e, e^-)	+1	0	0	(ν_e^c, e^+)	-1	0	0
(ν_μ, μ^-)	0	+1	0	(ν_μ^c, μ^+)	0	-1	0
(ν_τ, τ^-)	0	0	+1	(ν_τ^c, τ^+)	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

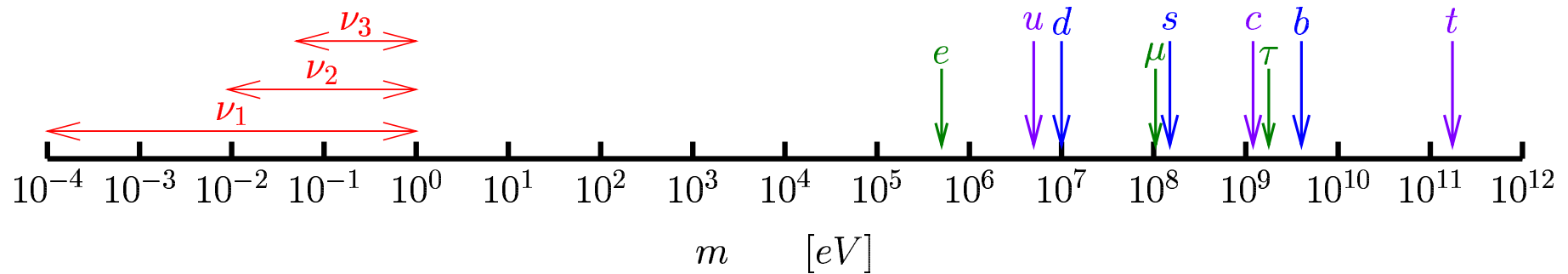
Dirac mass term $m^D \overline{\nu}_L \nu_R \Rightarrow (\overline{\nu}_{eL} \quad \overline{\nu}_{\mu L} \quad \overline{\nu}_{\tau L}) \begin{pmatrix} m_{ee}^D & m_{e\mu}^D & m_{e\tau}^D \\ m_{\mu e}^D & m_{\mu\mu}^D & m_{\mu\tau}^D \\ m_{\tau e}^D & m_{\tau\mu}^D & m_{\tau\tau}^D \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$

L_e, L_μ, L_τ are not conserved, but L is conserved $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Majorana mass term $m^M \overline{\nu}_L \nu_R^c \Rightarrow (\overline{\nu}_{eL} \quad \overline{\nu}_{\mu L} \quad \overline{\nu}_{\tau L}) \begin{pmatrix} m_{ee}^M & m_{e\mu}^M & m_{e\tau}^M \\ m_{\mu e}^M & m_{\mu\mu}^M & m_{\mu\tau}^M \\ m_{\tau e}^M & m_{\tau\mu}^M & m_{\tau\tau}^M \end{pmatrix} \begin{pmatrix} \nu_{eR}^c \\ \nu_{\mu R}^c \\ \nu_{\tau R}^c \end{pmatrix}$

L, L_e, L_μ, L_τ are not conserved $L(\nu_{\alpha R}^c) = -L(\nu_{\beta L}) \Rightarrow |\Delta L| = 2$

Majorana Neutrino Mass?



new high energy scale Λ connected with $|\Delta L| \neq 0$



known natural explanation of smallness of ν masses

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{c_5}{\Lambda} \Omega_5 + \frac{c_6}{\Lambda^2} \Omega_6 + \dots$$

[S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]

only one 5-D operator: $\Omega_5 \sim (HL)(HL) \rightarrow \Lambda_{\text{EW}}^2 \nu\nu$ $|\Delta L| = 2$

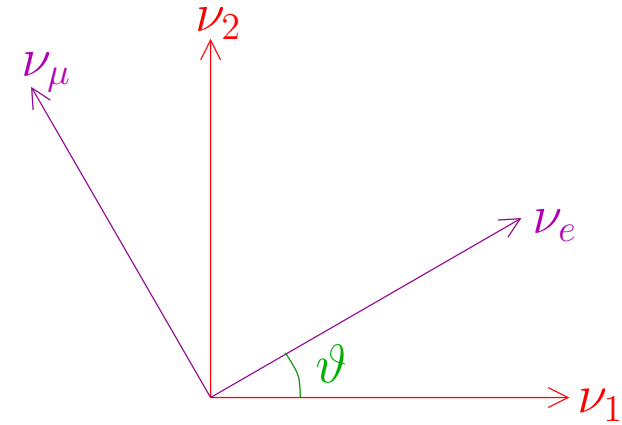
see-saw type generation of light Majorana neutrino masses

$$m_\nu \propto \frac{\Lambda_{\text{EW}}^2}{\Lambda} = \frac{\Lambda_{\text{EW}}}{\Lambda} \Lambda_{\text{EW}} \ll m_{\text{charged fermion}} \propto \Lambda_{\text{EW}} \sim 100 \text{ GeV}$$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^2 U_{\alpha k} |\nu_k\rangle \quad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix}$$

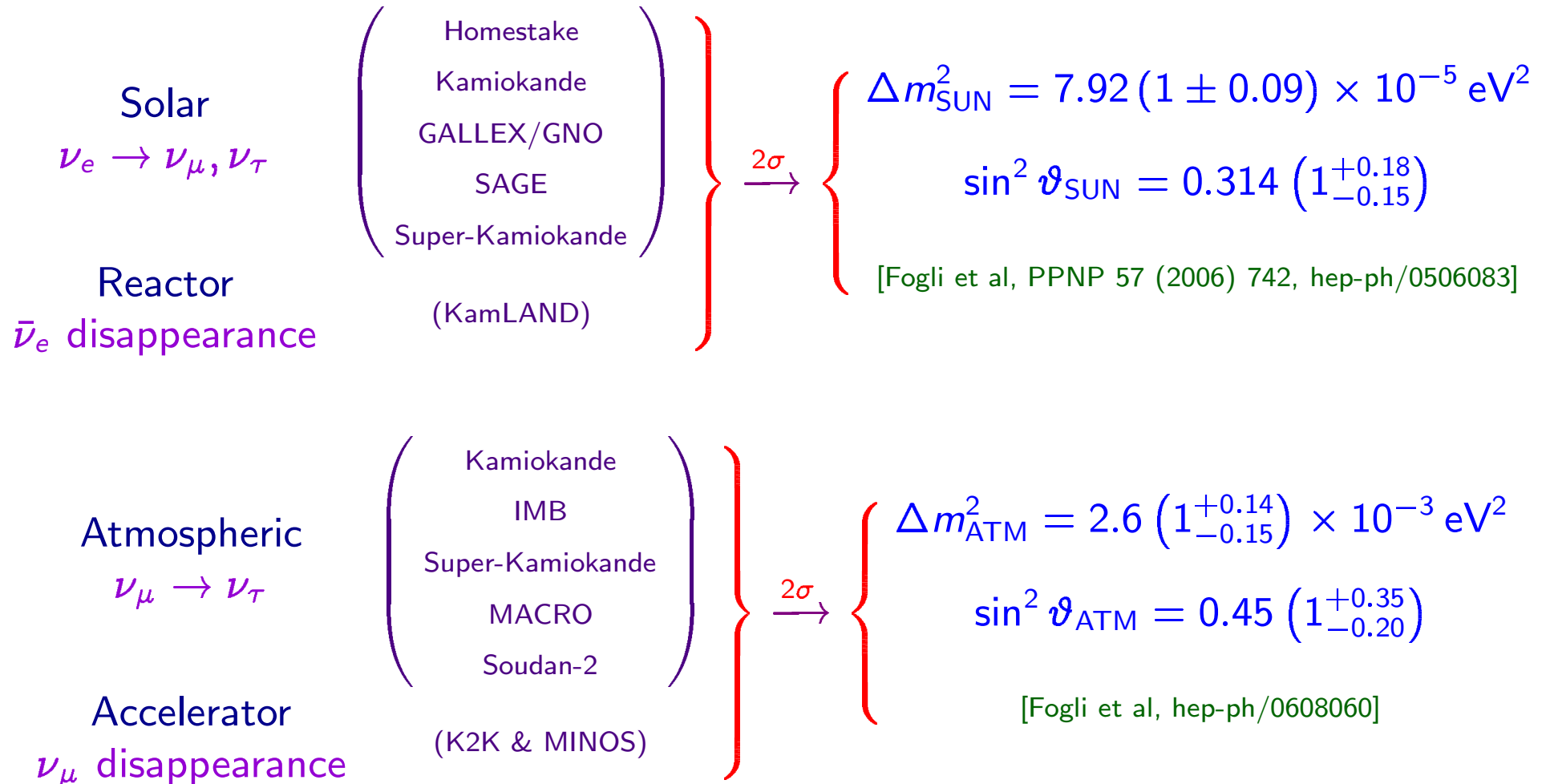
$$\begin{aligned} |\nu_e\rangle &= \cos \vartheta |\nu_1\rangle + \sin \vartheta |\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin \vartheta |\nu_1\rangle + \cos \vartheta |\nu_2\rangle \end{aligned}$$

$$\Delta m^2 \equiv \Delta m_{21}^2 \equiv m_2^2 - m_1^2$$

Transition Probability: $P_{\nu_e \rightarrow \nu_\mu} = P_{\nu_\mu \rightarrow \nu_e} = \sin^2 2\vartheta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$

Survival Probabilities: $P_{\nu_e \rightarrow \nu_e} = P_{\nu_\mu \rightarrow \nu_\mu} = 1 - P_{\nu_e \rightarrow \nu_\mu}$

Experimental Evidences of Neutrino Oscillations



Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^3 U_{\alpha k} \nu_{kL} \quad (\alpha = e, \mu, \tau)$$

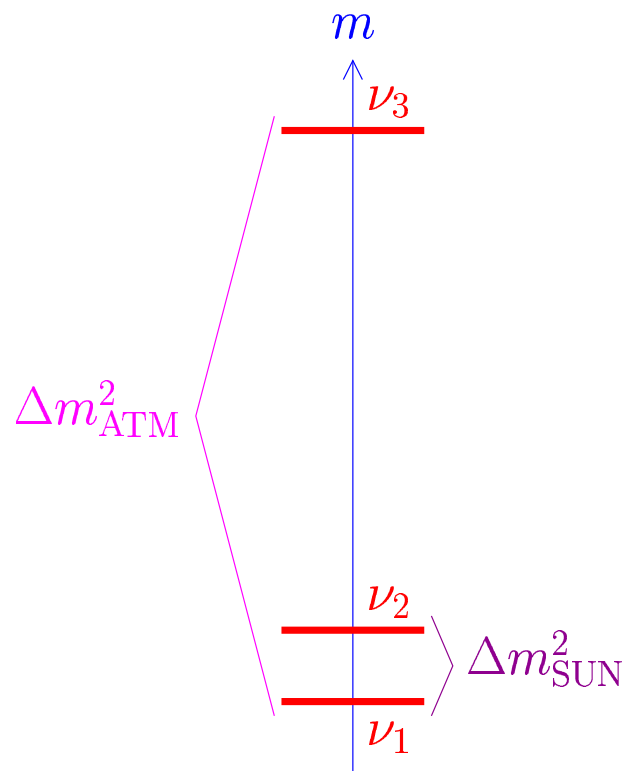
three flavor fields ν_e, ν_μ, ν_τ

three massive fields ν_1, ν_2, ν_3

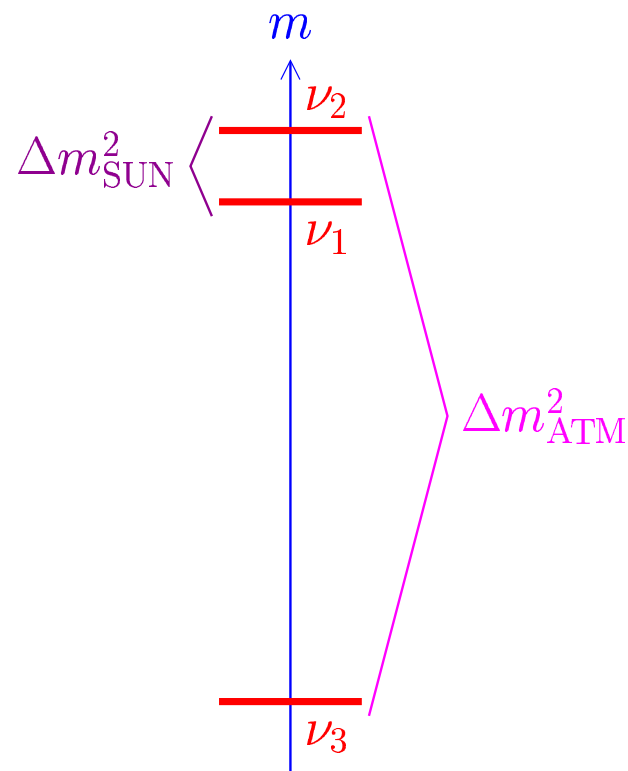
$$\Delta m_{\text{SUN}}^2 = \Delta m_{21}^2 \simeq 8.0 \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{\text{ATM}}^2 \simeq |\Delta m_{31}^2| \simeq |\Delta m_{32}^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2$$

Allowed Three-Neutrino Schemes



"normal"



"inverted"

different signs of $\Delta m_{31}^2 \simeq \Delta m_{32}^2$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\Delta m_{21}^2 \ll |\Delta m_{31}^2|$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

SUN →
↑
 ATM

$$\text{CHOOZ: } \begin{cases} \Delta m_{\text{CHOOZ}}^2 = \Delta m_{31}^2 = \Delta m_{\text{ATM}}^2 \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$$

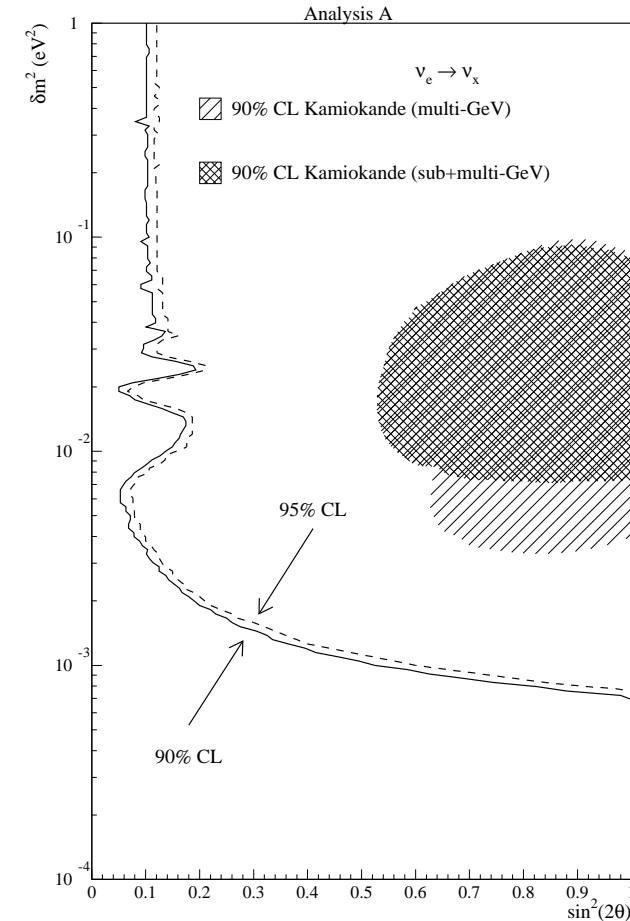
$$|U_{e3}|^2 < 5 \times 10^{-2} \quad (99.73\% \text{ C.L.})$$

[Fogli et al., PRD 66 (2002) 093008]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

TWO-NEUTRINO SOLAR and ATMOSPHERIC ν OSCILLATIONS ARE OK!

$$\sin^2 \vartheta_{\text{SUN}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \simeq |U_{e2}|^2 \quad \sin^2 \vartheta_{\text{ATM}} = |U_{\mu 3}|^2$$



[CHOOZ, PLB 466 (1999) 415]

see also [Palo Verde, PRD 64 (2001) 112001]

[Bilenky, C.G, PLB 444 (1998) 379]

[Guo, Xing, PRD 67 (2003) 053002]

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

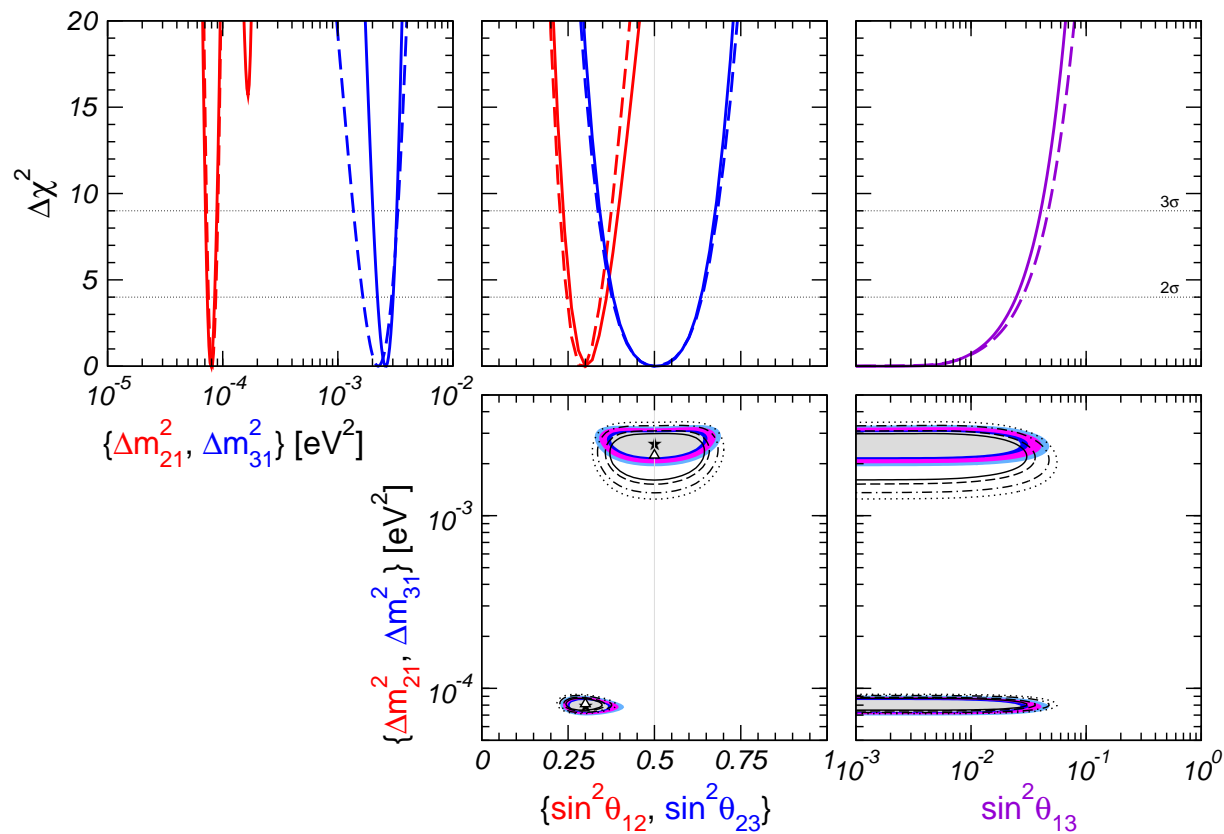
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$\vartheta_{23} \simeq \vartheta_{\text{ATM}}$ $\vartheta_{13} = \vartheta_{\text{CHOOZ}}$ $\vartheta_{12} = \vartheta_{\text{SUN}}$ $\beta\beta_{0\nu}$

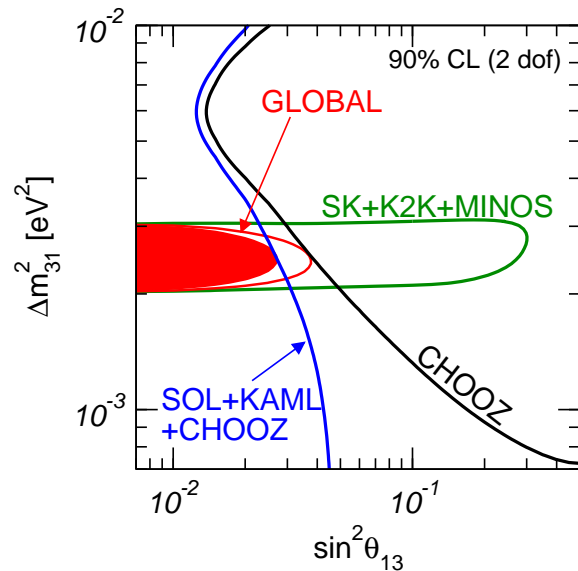
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\text{CHOOZ} + \text{SK} + \text{MINOS} \implies \sin^2 \vartheta_{\text{CHOOZ}} = 0.008_{-0.008}^{+0.023} @ 2\sigma$$

[Fogli et al, hep-ph/0608060]



[Maltoni, Schwetz, Tortola, Valle, hep-ph/0405172 v5]



$$P_{ee}^{\text{SUN}} = c_{13}^4 P_{ee}^{(2)}(\Delta m_{21}^2, \vartheta_{12}, c_{13}^2 V) + s_{13}^4$$

$$\sin^2 \vartheta_{13} < 0.020 \text{ (90\% CL)}, 0.041 \text{ (3}\sigma\text{)}$$

[Schwetz, hep-ph/0606060]

Bilarge Mixing

$$|U_{e3}|^2 \ll 1$$

$$U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S} c_{\vartheta_A} & c_{\vartheta_S} c_{\vartheta_A} & s_{\vartheta_A} \\ s_{\vartheta_S} s_{\vartheta_A} & -c_{\vartheta_S} s_{\vartheta_A} & c_{\vartheta_A} \end{pmatrix} \implies \begin{cases} \nu_e = c_{\vartheta_S} \nu_1 + s_{\vartheta_S} \nu_2 \\ \nu_a^{(S)} = -s_{\vartheta_S} \nu_1 + c_{\vartheta_S} \nu_2 \\ = c_{\vartheta_A} \nu_\mu - s_{\vartheta_A} \nu_\tau \end{cases}$$

$$\sin^2 2\vartheta_A \simeq 1 \implies \vartheta_A \simeq \frac{\pi}{4} \implies U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S}/\sqrt{2} & c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \\ s_{\vartheta_S}/\sqrt{2} & -c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\text{Solar } \nu_e \rightarrow \nu_a^{(S)} \simeq \frac{1}{\sqrt{2}} (\nu_\mu - \nu_\tau)$$

$$\frac{\Phi_{\text{CC}}^{\text{SNO}}}{\Phi_{\nu_e}^{\text{SSM}}} \simeq \frac{1}{3} \implies \Phi_{\nu_e} \simeq \Phi_{\nu_\mu} \simeq \Phi_{\nu_\tau} \text{ for } E \gtrsim 6 \text{ MeV}$$

$$\sin^2 \vartheta_S \simeq \frac{1}{3} \implies U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-Bimaximal Mixing

[Harrison, Perkins, Scott, hep-ph/0202074]

Global Fit of Oscillation Data: Bilarge Mixing

$$\Delta m_{21}^2 = 7.92 (1 \pm 0.09) \times 10^{-5} \text{ eV}^2 \quad \sin^2 \vartheta_{12} = 0.314 (1_{-0.15}^{+0.18})$$

$$|\Delta m_{31}^2| = 2.6 (1_{-0.15}^{+0.14}) \times 10^{-3} \text{ eV}^2 \quad \sin^2 \vartheta_{23} = 0.45 (1_{-0.20}^{+0.35})$$

$$\sin^2 \vartheta_{13} = 0.008_{-0.008}^{+0.023}$$

[Fogli et al, hep-ph/0608060]

$$|U|_{\text{bf}} \simeq \begin{pmatrix} 0.82 & 0.56 & 0.09 \\ 0.37 - 0.47 & 0.58 - 0.65 & 0.67 \\ 0.32 - 0.43 & 0.52 - 0.59 & 0.74 \end{pmatrix}$$

$$|U|_{2\sigma} \simeq \begin{pmatrix} 0.78 - 0.86 & 0.51 - 0.61 & 0.00 - 0.18 \\ 0.21 - 0.57 & 0.41 - 0.74 & 0.59 - 0.78 \\ 0.19 - 0.56 & 0.39 - 0.72 & 0.62 - 0.80 \end{pmatrix}$$

future: measure $\vartheta_{13} \neq 0 \implies$ CP violation, matter effects, mass hierarchy

Future Reactor Experiments

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \simeq 1 - \sin^2 2\vartheta_{13} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) \quad \langle E \rangle \simeq 3.6 \text{ MeV}$$

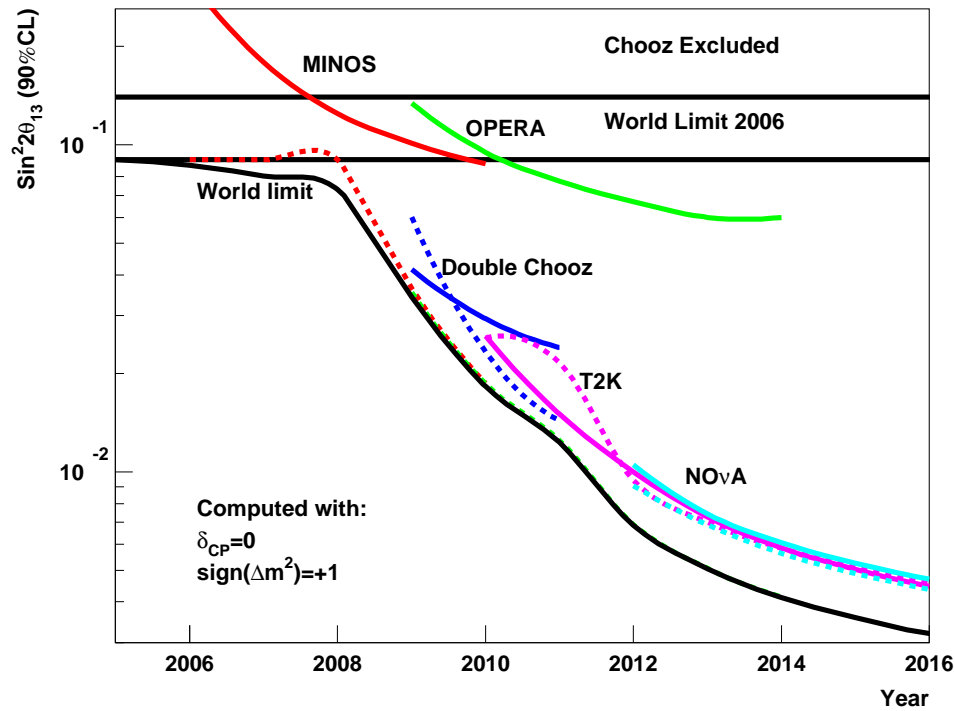
Exp.	Place	L (km)	Start (yr)	Duration (yr)	Sensitivity ($\sin^2 2\vartheta_{13}$)
Double-CHOOZ Far Detector	France	1.0	2008	1.5	0.06
Double-CHOOZ Near+Far	France	1.0	2010	3	0.03
KASKA	Japan		2010	3	0.02
RENO	Korea	1.5	2009	3	0.02
Daya Bay Near+Mid	China	1.0	2008	1	0.03
Daya Bay Near+Far	China	1.9	2010	3	0.01

Future Accelerator Experiments: Off-Axis

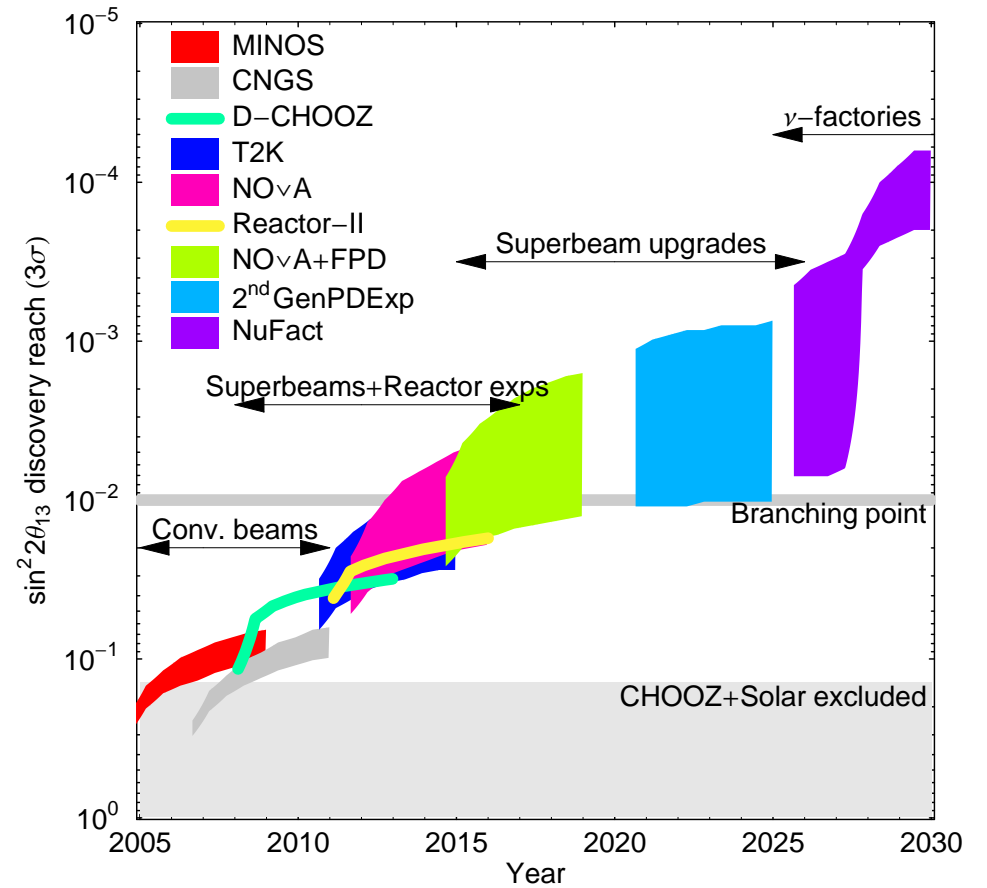
$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} \simeq \sin^2 \vartheta_{23} \sin^2 2\vartheta_{13} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4 E} \right)$$

Exp.	Place	$\langle E \rangle$ (GeV)	L (km)	Start (yr)	Duration (yr)	Sensitivity ($\sin^2 2\vartheta_{13}$)
T2K	Japan	0.7	295	2009	4	0.01
NO ν A	USA	2	810	2011	4	0.01

The Hunt for ϑ_{13}



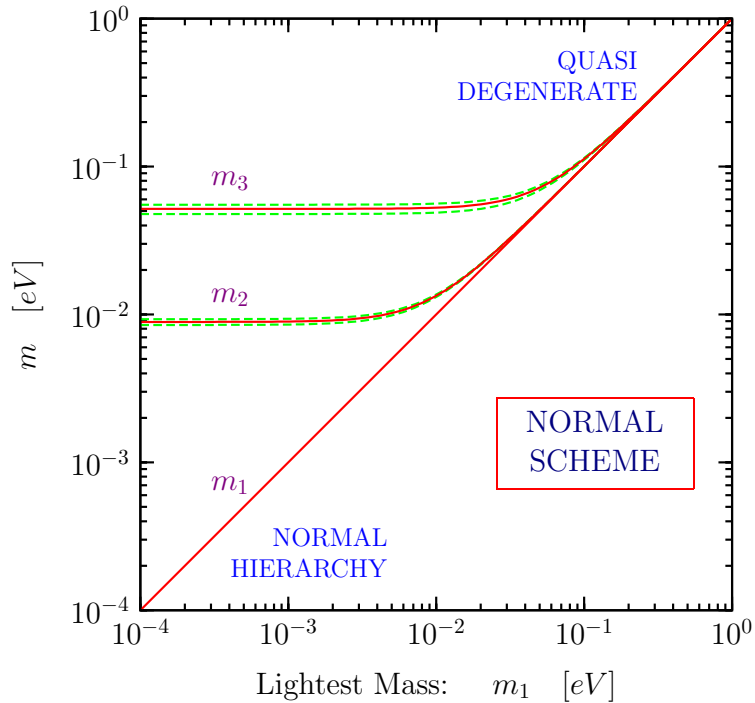
[Blondel et al, hep-ph/0606111]



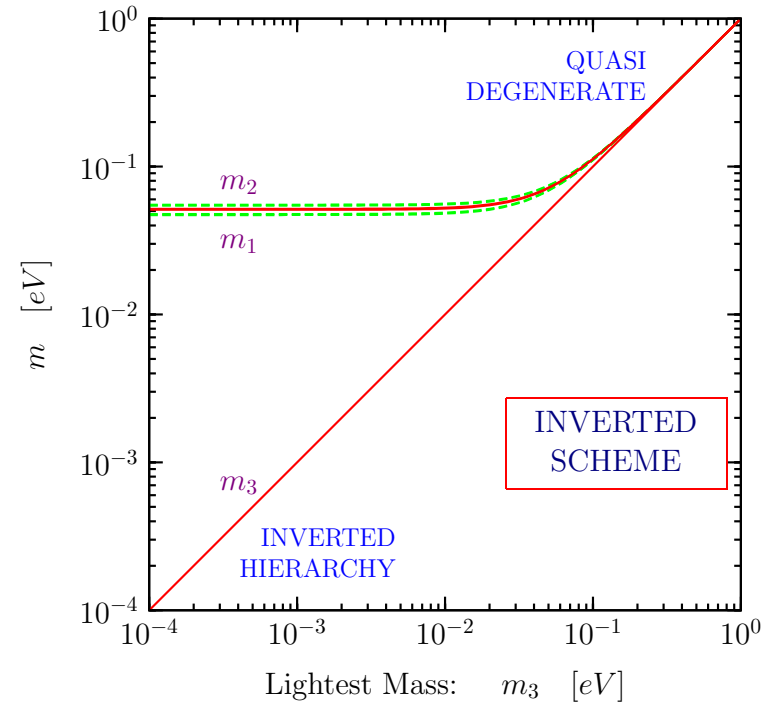
[Albrow et al, hep-ex/0509019]

Absolute Scale of Neutrino Masses

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SUN}}^2$$

$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$

$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2} \text{ eV}$

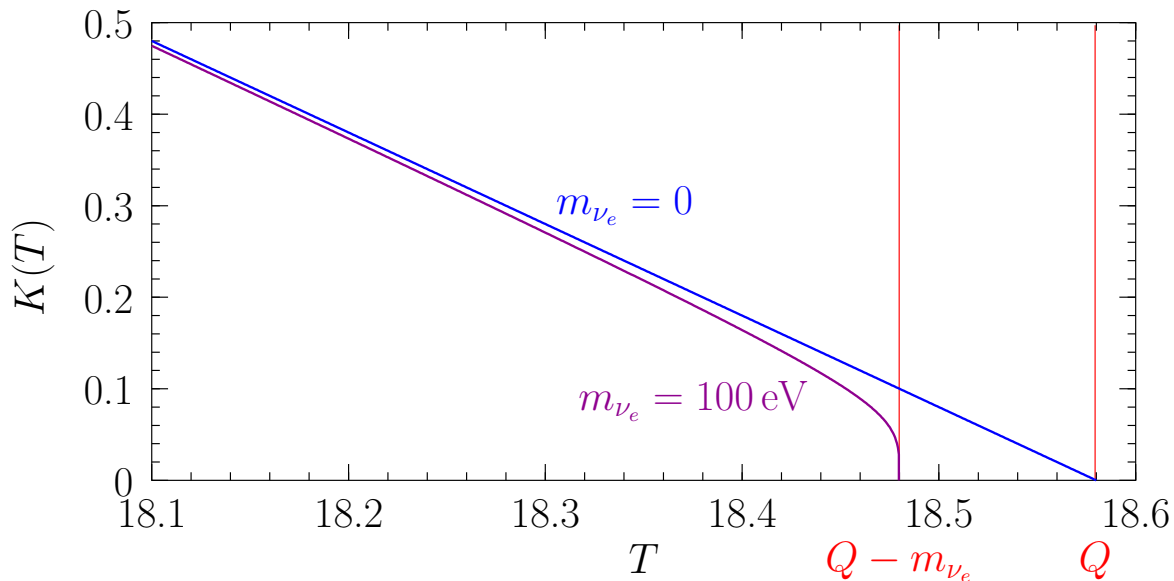
Tritium Beta-Decay

$$\underline{{}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e} \quad \frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) pE (Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2}$$

$$Q = M_{{}^3\text{H}} - M_{{}^3\text{He}} - m_e = 18.58 \text{ keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) pE}} = \left[(Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2} \right]^{1/2}$$



$$m_{\nu_e} < 2.2 \text{ eV} \quad (95\% \text{ C.L.})$$

Mainz & Troitsk

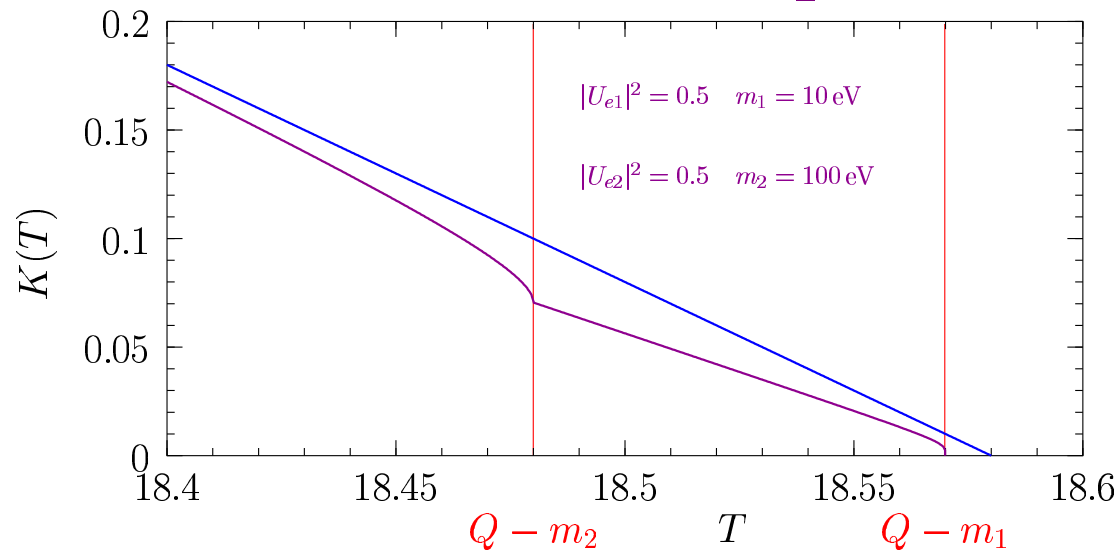
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2010)

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_{\nu_e} \simeq 0.2 \text{ eV}$

$$\text{Neutrino Mixing} \implies K(T) = \left[(Q - T) \sum_k |U_{ek}|^2 \sqrt{(Q - T)^2 - m_k^2} \right]^{1/2}$$



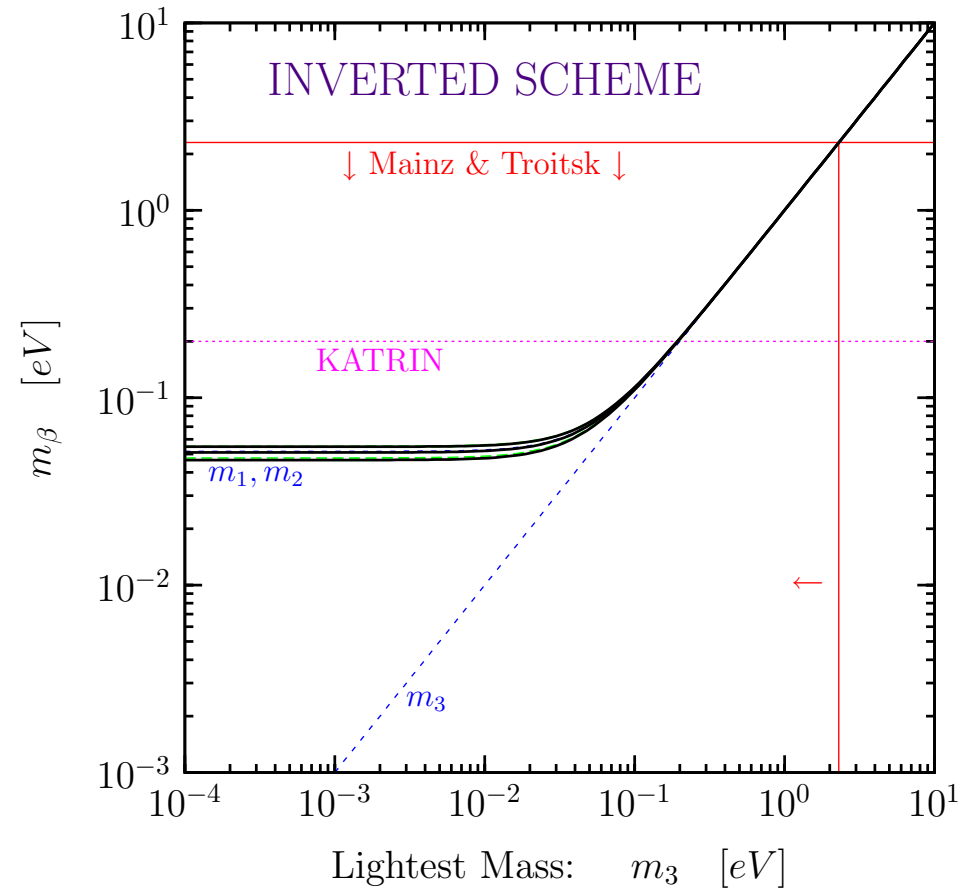
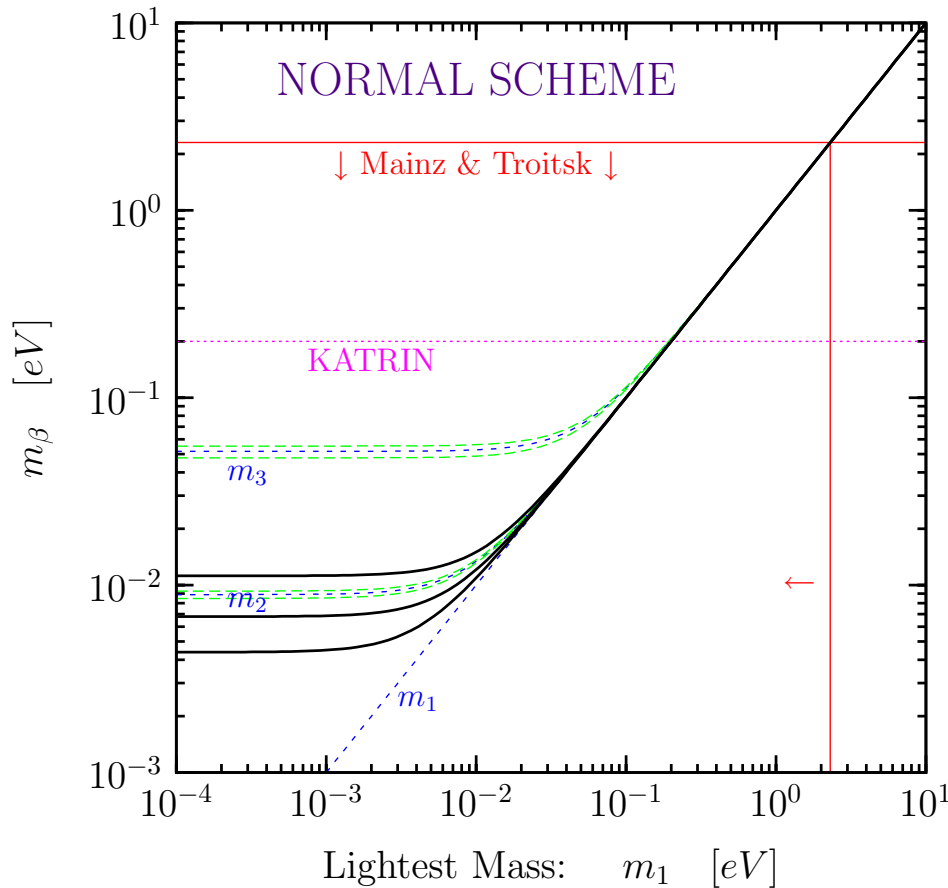
analysis of data is different from the no-mixing case:
 $2N - 1$ parameters
 $\left(\sum_k |U_{ek}|^2 = 1 \right)$

if experiment is not sensitive to masses ($m_k \ll Q - T$)

effective mass: $m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$

$$\begin{aligned} K^2 &= (Q - T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q - T)^2}} \simeq (Q - T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q - T)^2} \right] \\ &= (Q - T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q - T)^2} \right] \simeq (Q - T) \sqrt{(Q - T)^2 - m_\beta^2} \end{aligned}$$

$$m_\beta^2 = |U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2$$

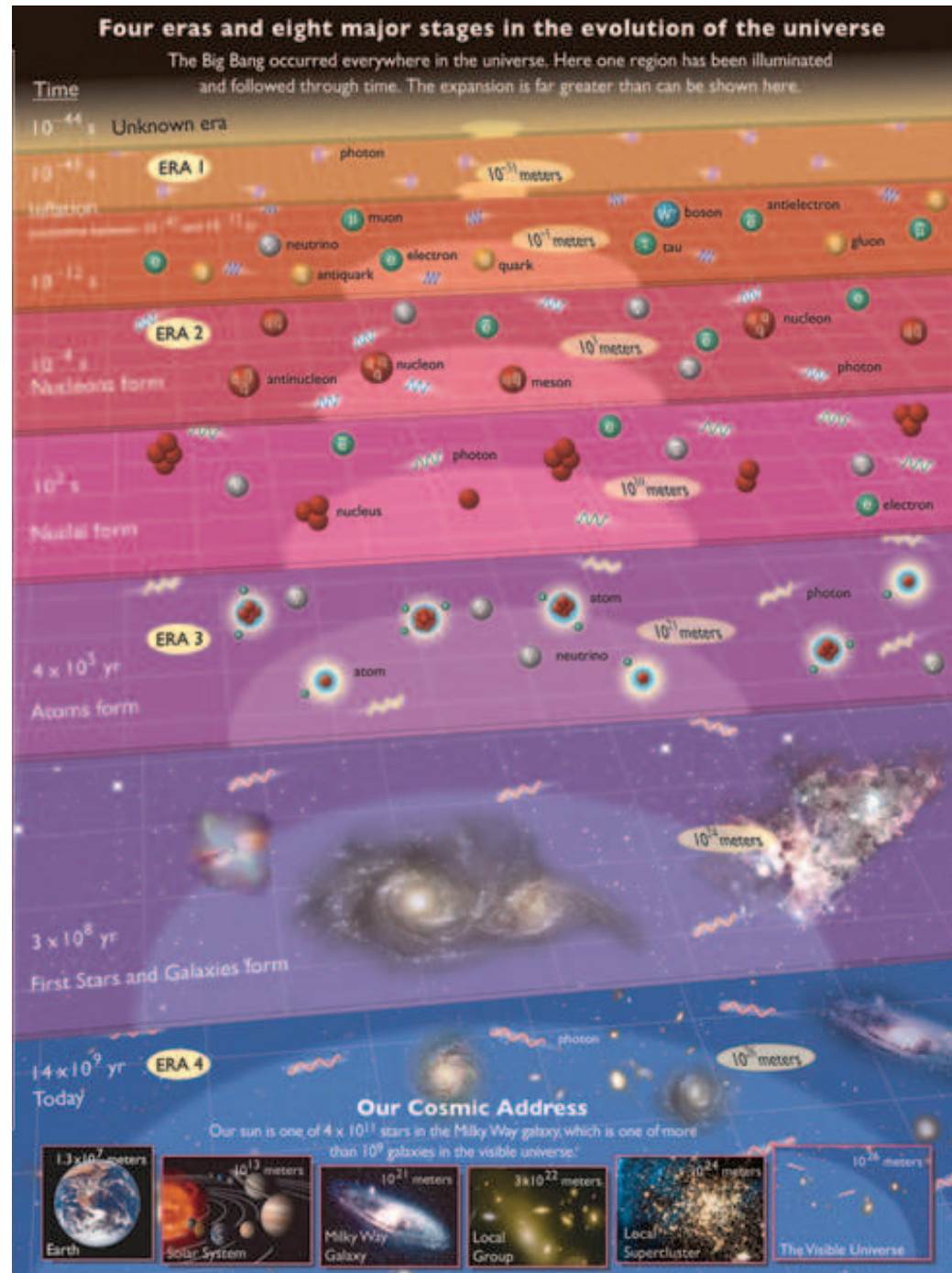


Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

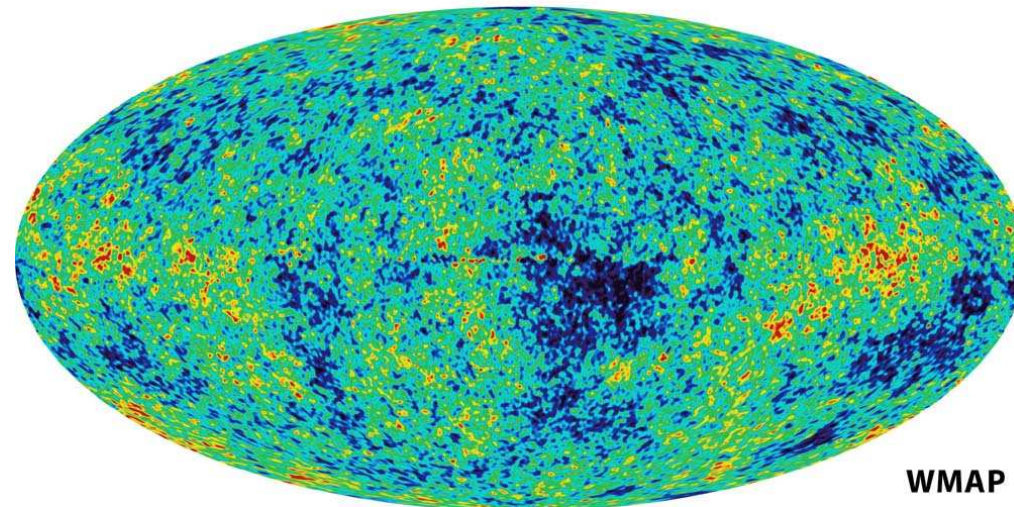
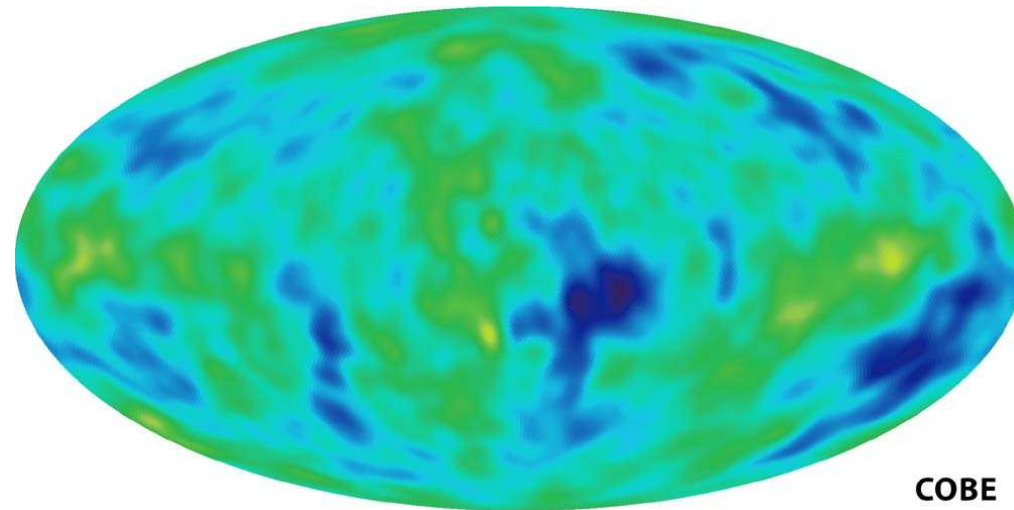
FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2} \text{ eV} \implies$ NORMAL HIERARCHY

Cosmological Bound on Neutrino Masses

[CPEP, <http://www.cpepweb.org/>]

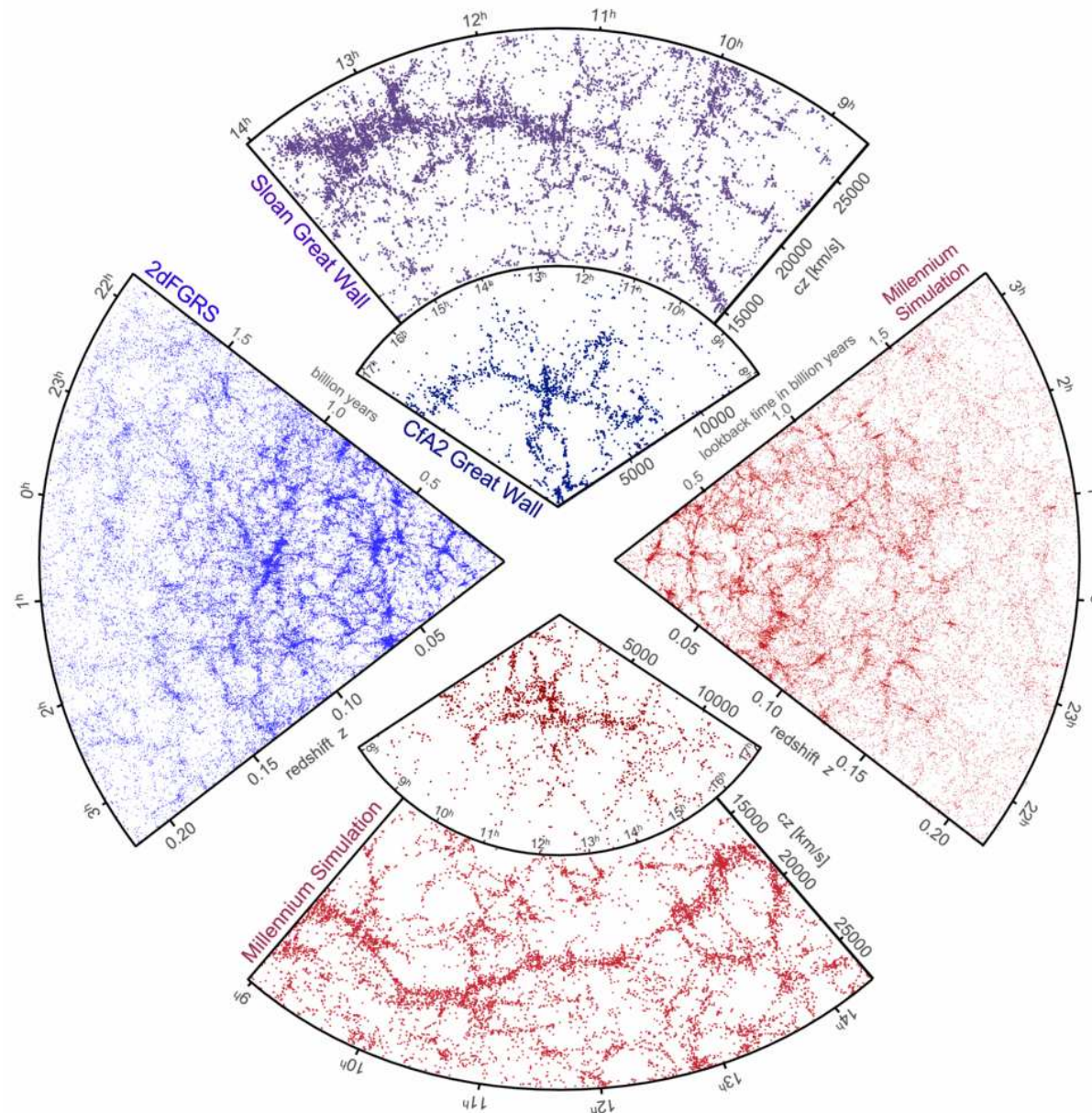


WMAP (Wilkinson Microwave Anisotropy Probe)



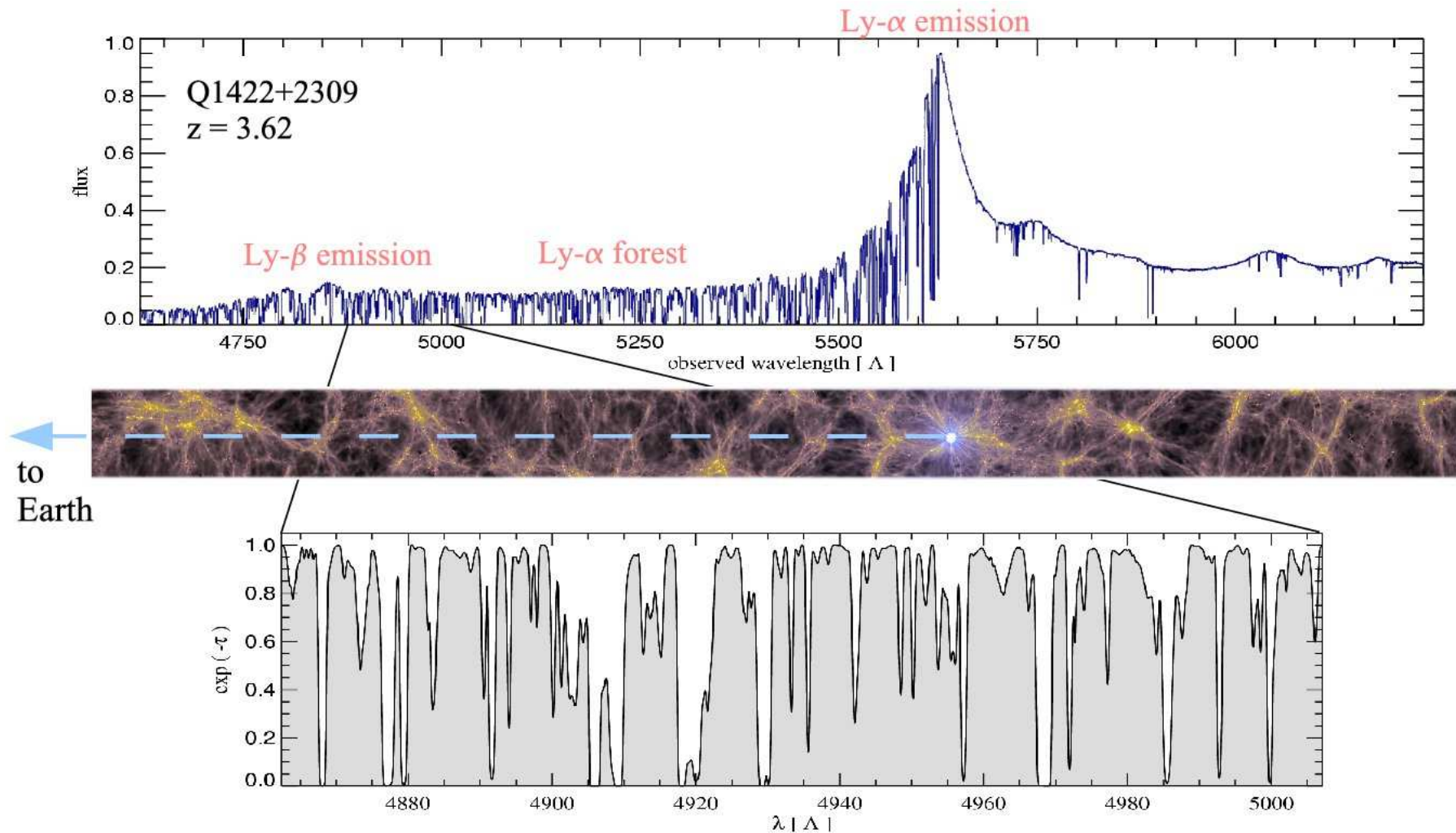
[WMAP, <http://map.gsfc.nasa.gov>]

Galaxy Redshift Surveys



[Springel, Frenk, White, astro-ph/0604561]

Lyman-alpha Forest



[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α , β , γ wavelengths: $\lambda_{\alpha}^0 = 1215.67 \text{ \AA}$, $\lambda_{\beta}^0 = 1025.72 \text{ \AA}$, $\lambda_{\gamma}^0 = 972.54 \text{ \AA}$

Lyman- α forest: The region in which only Ly α photons can be absorbed: $[(1 + z_q)\lambda_{\beta}^0, (1 + z_q)\lambda_{\alpha}^0]$

Relic Neutrinos

neutrinos are in equilibrium in primeval plasma through weak interaction reactions



weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \implies T_{\text{dec}} \sim 1 \text{ MeV}$$

neutrino decoupling

Relic Neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945 \text{ K} \implies k T_\nu \simeq 1.676 \times 10^{-4} \text{ eV}$
($T_\gamma = 2.725 \pm 0.001 \text{ K}$)

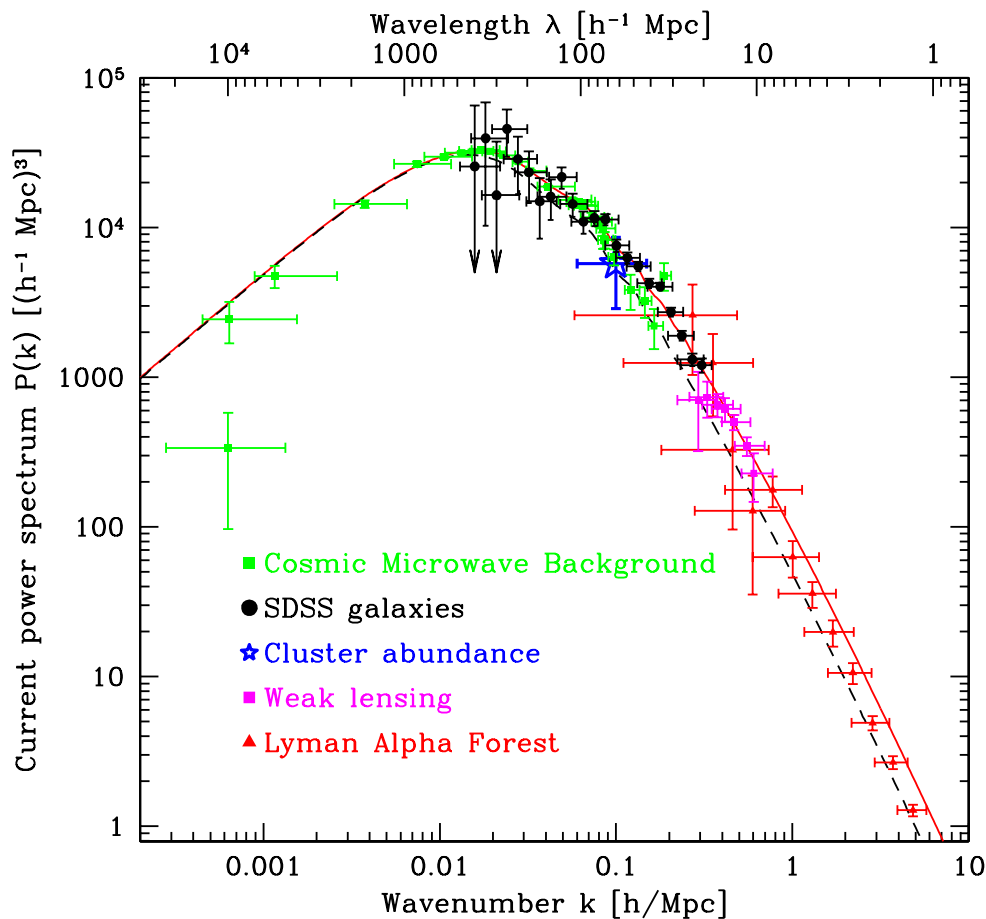
number density: $n_f = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_f T_f^3 \implies n_{\nu_k, \bar{\nu}_k} \simeq 0.1827 T_\nu^3 \simeq 112 \text{ cm}^{-3}$

density contribution: $\Omega_k = \frac{n_{\nu_k, \bar{\nu}_k} m_k}{\rho_c} \simeq \frac{1}{h^2} \frac{m_k}{94.14 \text{ eV}} \implies \Omega_\nu h^2 = \frac{\sum_k m_k}{94.14 \text{ eV}}$

($\rho_c = \frac{3H^2}{8\pi G_N}$) [Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \implies \quad \sum_k m_k \lesssim 14 \text{ eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat Λ CDM model
 $(\Omega_M^0 = 0.28, h = 0.72, \Omega_B^0/\Omega_M^0 = 0.16)$

Dashed Curve: $\sum_{k=1}^3 m_k = 1 \text{ eV}$

hot dark matter
prevents early galaxy formation

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8 \frac{\Omega_\nu}{\Omega_m}$$

$$\approx -0.8 \left(\frac{\sum_k m_k}{1 \text{ eV}} \right) \left(\frac{0.1}{\Omega_m h^2} \right)$$

for

$$k \gtrsim k_{\text{nr}} \approx 0.026 \sqrt{\frac{m_\nu}{1 \text{ eV}}} \sqrt{\Omega_m} h \text{ Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP, AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SNIa $\implies \Lambda$ CDM

$$T_0 = 13.7 \pm 0.1 \text{ Gyr} \quad h = 0.71^{+0.04}_{-0.03}$$
$$\Omega_0 = 1.02 \pm 0.02 \quad \Omega_B h^2 = 0.0224 \pm 0.0009 \quad \Omega_M h^2 = 0.135^{+0.008}_{-0.009}$$

$$\Omega_\nu h^2 < 0.0076 \quad (95\% \text{ conf.}) \implies \sum_{k=1}^3 m_k < 0.71 \text{ eV}$$

WMAP, astro-ph/0603449

Flat Λ CDM (WMAP+HST: $\Omega_0 = 1.010^{+0.016}_{-0.009}$, $\Omega_\Lambda = 0.72 \pm 0.04$)

$$\sum_{k=1}^3 m_k < \begin{cases} 2.0 \text{ eV} & \text{WMAP} \\ 0.91 \text{ eV} & \text{WMAP+SDSS} \\ 0.87 \text{ eV} & \text{WMAP+2dFGRS} \\ 0.68 \text{ eV} & \text{CMB+LSS+SNIa} \end{cases} \quad (95\% \text{ conf.})$$

Flat Λ CDM

$$\sum_{k=1}^3 m_k < \begin{cases} 0.70 \text{ eV} & \text{CMB+LSS+SNIa} \\ 0.48 \text{ eV} & \text{CMB+LSS+SNIa+BAO} \\ 0.27 \text{ eV} & \text{CMB+LSS+SNIa+BAO+Ly}\alpha \end{cases} \quad (95\% \text{ conf.})$$

Seljak, Slosar, McDonald, astro-ph/0604335

Flat Λ CDM

CMB+LSS+SNIa+BAO+Ly α

$$\sum_{k=1}^3 m_k < 0.17 \text{ eV} \quad (95\% \text{ conf.})$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, Slosar, hep-ph/0608060

Flat Λ CDM

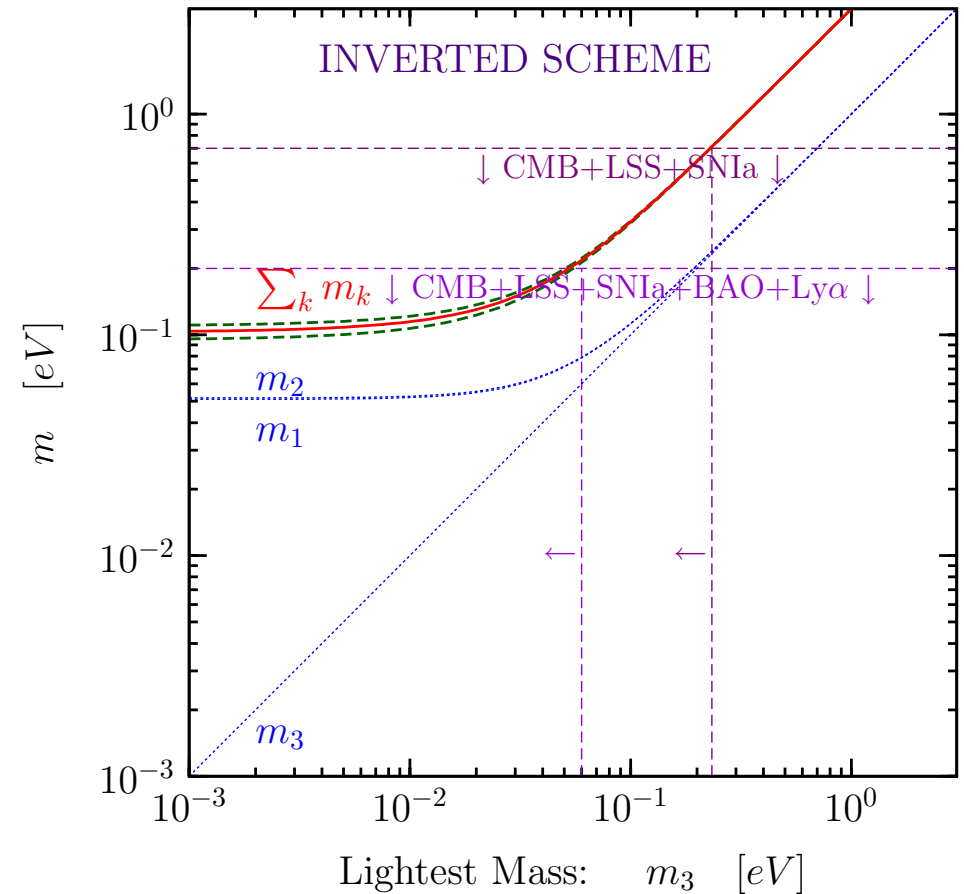
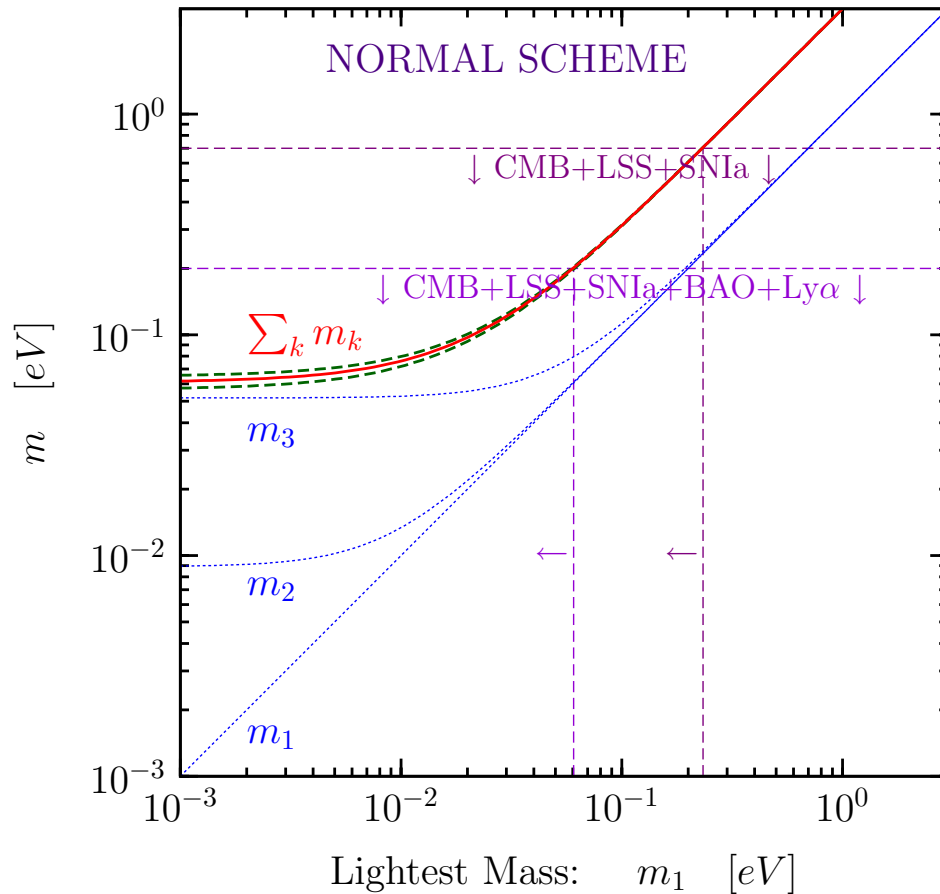
$$\sum_{k=1}^3 m_k < \begin{cases} 0.75 \text{ eV} & \text{CMB+LSS+SNIa} \\ 0.58 \text{ eV} & \text{CMB+LSS+SNIa+BAO} \\ 0.17 \text{ eV} & \text{CMB+LSS+SNIa+BAO+Ly}\alpha \end{cases} \quad (95\% \text{ conf.})$$

$$\sum_{k=1}^3 m_k \lesssim 0.7 \text{ eV} \quad (\sim 2\sigma)$$

CMB+LSS+SN Ia

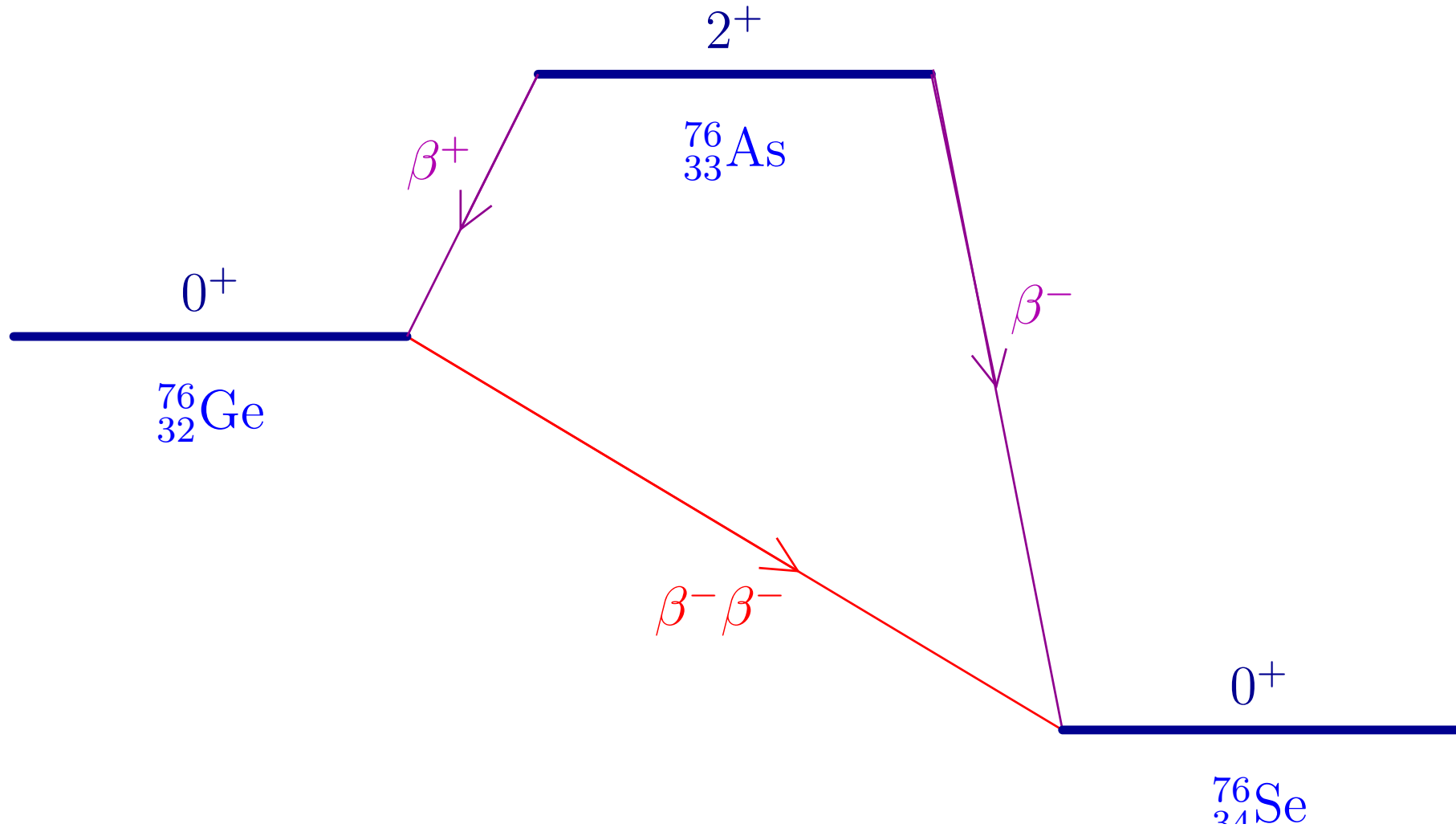
$$\sum_{k=1}^3 m_k \lesssim 0.2 \text{ eV} \quad (\sim 2\sigma)$$

CMB+LSS+SN Ia+BAO+Ly α



FUTURE: IF $\sum_{k=1}^3 m_k \lesssim 9 \times 10^{-2} \text{ eV} \implies$ NORMAL HIERARCHY

Neutrinoless Double-Beta Decay

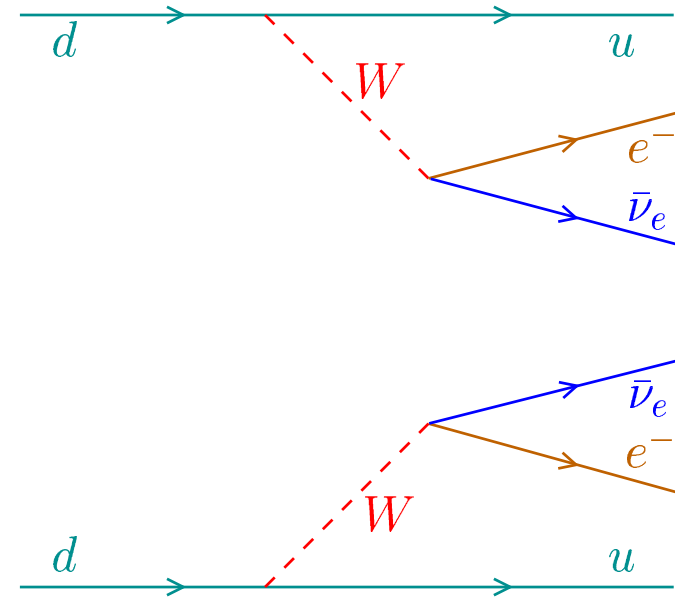


Two-Neutrino Double- β Decay: $\Delta L = 0$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu} |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model



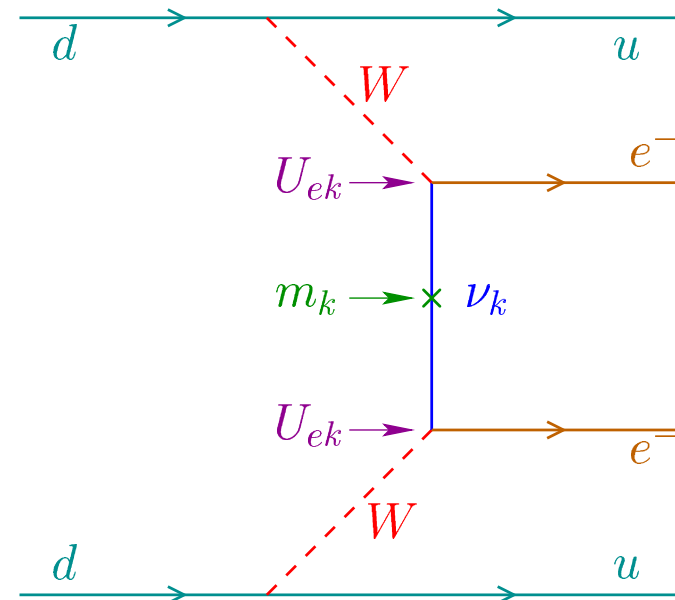
Neutrinoless Double- β Decay: $\Delta L = 2$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu} |\mathcal{M}_{0\nu}|^2 |m_{\beta\beta}|^2$$

effective
Majorana
mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k$$

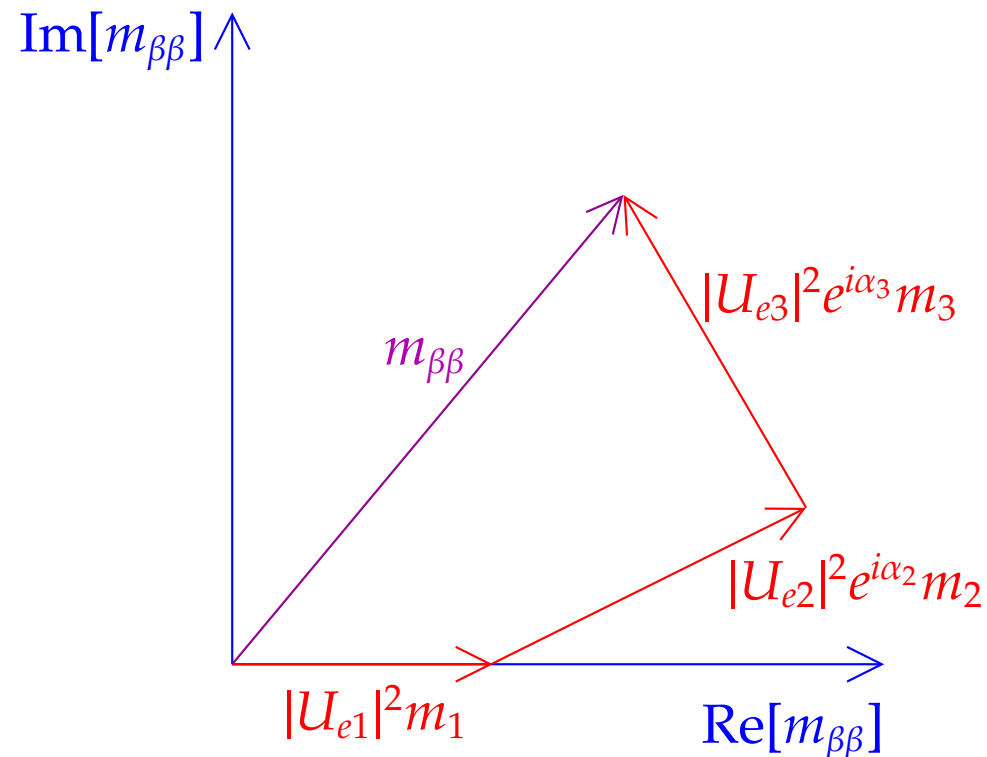


Effective Majorana Neutrino Mass

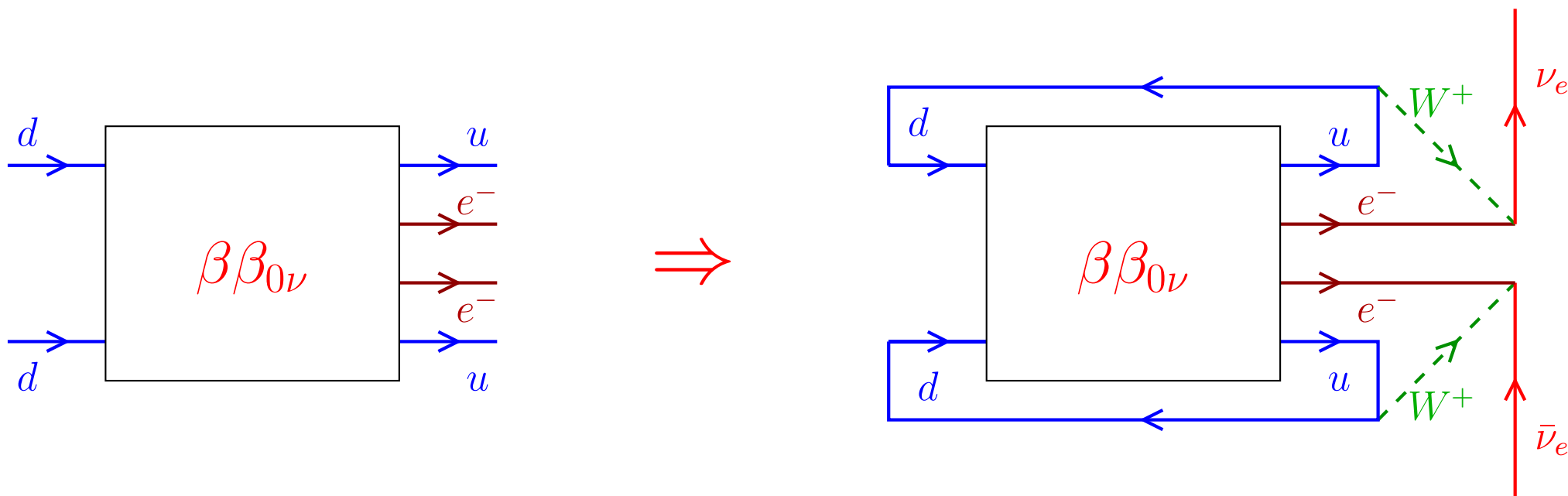
$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k \quad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3$$

$$\alpha_2 = 2\lambda_2 \quad \alpha_3 = 2(\lambda_3 - \delta_{13})$$



Majorana Neutrino Mass $\Leftrightarrow \beta\beta_{0\nu}$ Decay



[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

Majorana Mass Term

$$\mathcal{L}_L^M = -\frac{1}{2} m \left(\bar{\nu}_L^c \nu_L + \bar{\nu}_L \nu_L^c \right) = \frac{1}{2} m \left(\nu_L^T C^\dagger \nu_L + \nu_L^\dagger C \nu_L^* \right)$$

two conditions: $\left\{ \begin{array}{l} u, d, e \text{ are massive} \\ \text{standard left-handed weak interaction exists} \end{array} \right.$
 cancellations with other diagrams are very unlikely
 (unstable under perturbations)

Experimental Bound

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$T_{1/2}^{0\nu} > 1.9 \times 10^{25} \text{ y} \quad (90\% \text{ C.L.}) \implies |m_{\beta\beta}| \lesssim 0.32 - 1.0 \text{ eV}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$T_{1/2}^{0\nu} > 1.57 \times 10^{25} \text{ y} \quad (90\% \text{ C.L.}) \implies |m_{\beta\beta}| \lesssim 0.33 - 1.35 \text{ eV}$$

CUORICINO (^{130}Te) [PRL 95 (2005) 142501]

$$T_{1/2}^{0\nu} > 1.8 \times 10^{24} \text{ y} \quad (90\% \text{ C.L.}) \implies |m_{\beta\beta}| \lesssim 0.2 - 1.1 \text{ eV}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$$T_{1/2}^{0\nu} > 4.6 \times 10^{23} \text{ y} \quad (90\% \text{ C.L.}) \implies |m_{\beta\beta}| \lesssim 0.7 - 2.8 \text{ eV}$$

FUTURE EXPERIMENTS

NEMO 3, CUORICINO, COBRA, XMASS, CAMEO, CANDLES

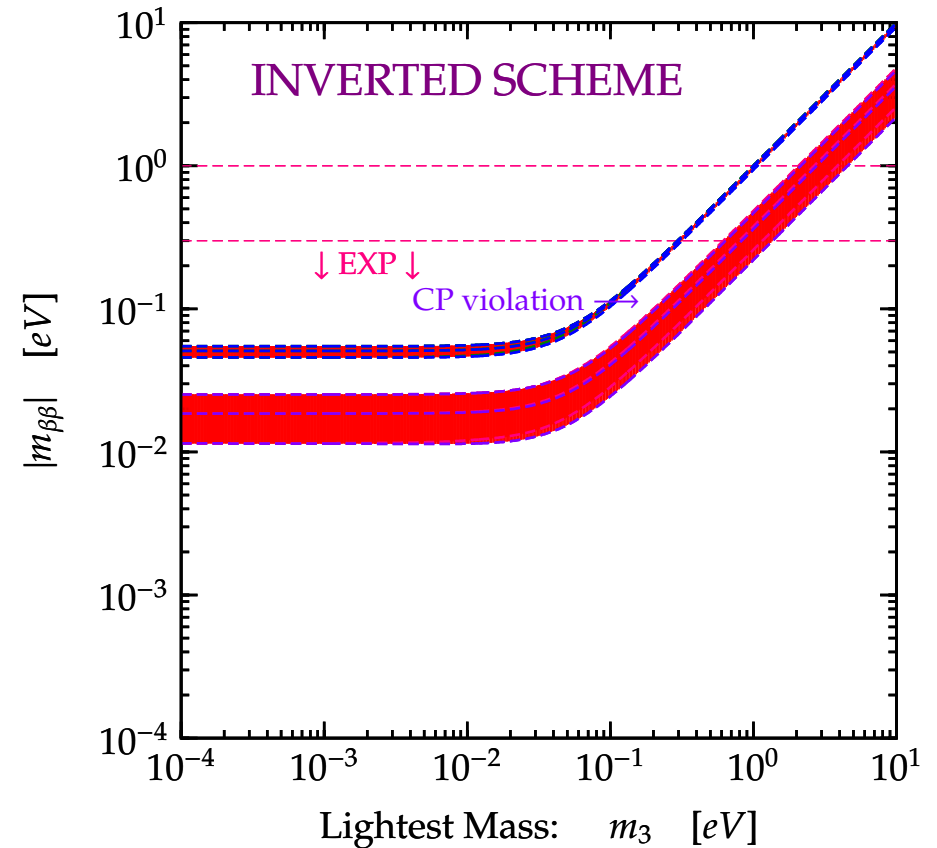
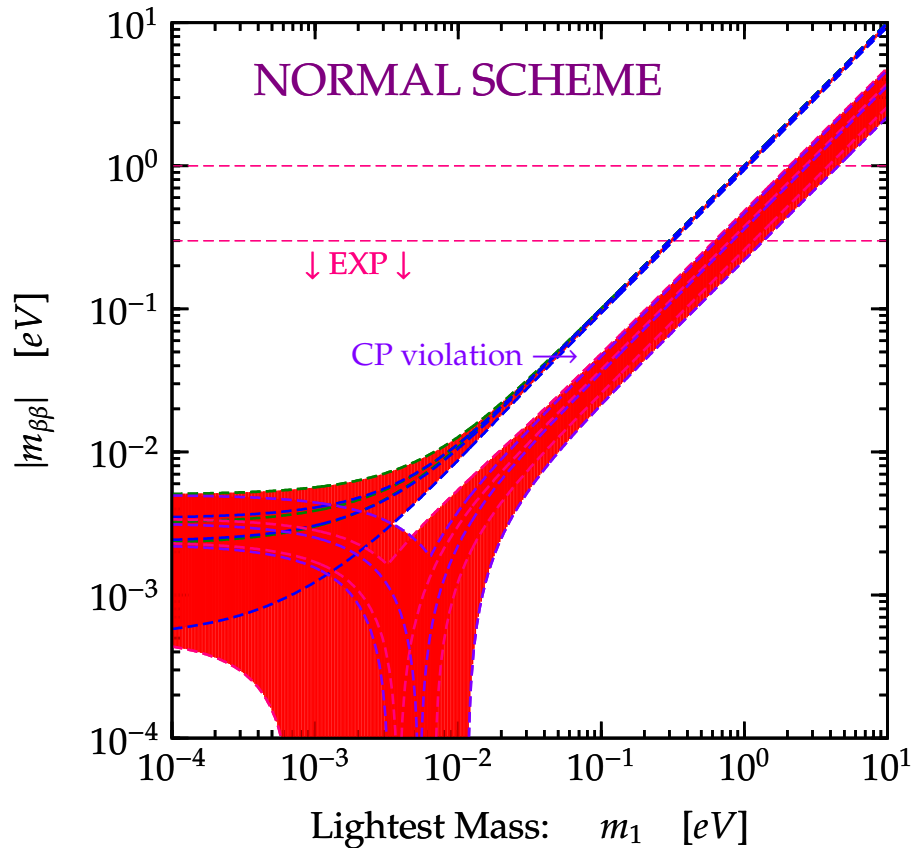
$$|m_{\beta\beta}| \sim \text{few } 10^{-1} \text{ eV}$$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$$|m_{\beta\beta}| \sim \text{few } 10^{-2} \text{ eV}$$

Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3$$

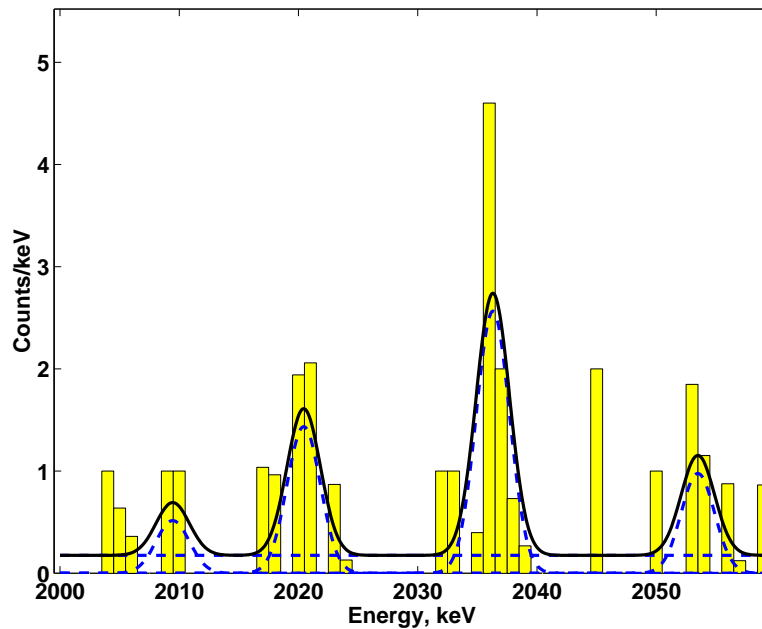


FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2} \text{ eV} \implies$ NORMAL HIERARCHY

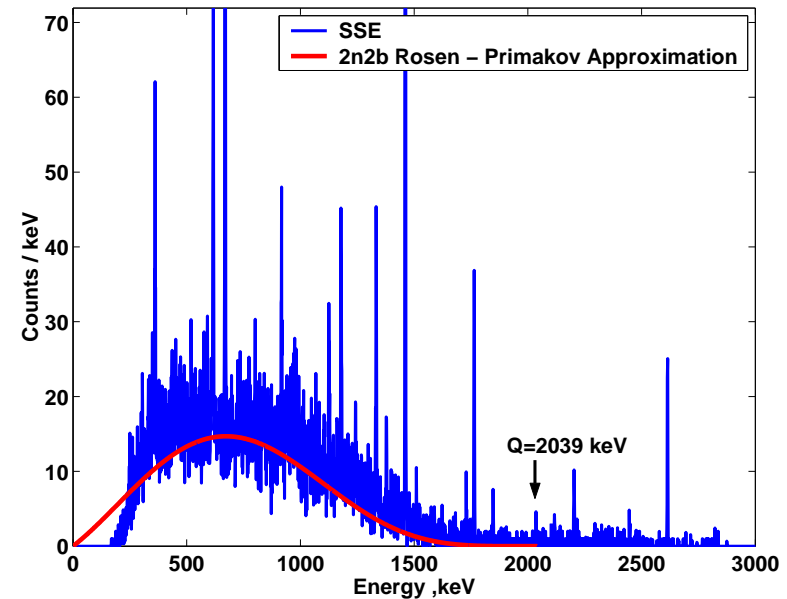
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]

$$T_{1/2}^{0\nu\text{bf}} = 1.19 \times 10^{25} \text{ y} \quad T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25} \text{ y} (3\sigma) \quad 4.2\sigma \text{ evidence}$$



pulse-shape selected spectrum



3.8 σ evidence

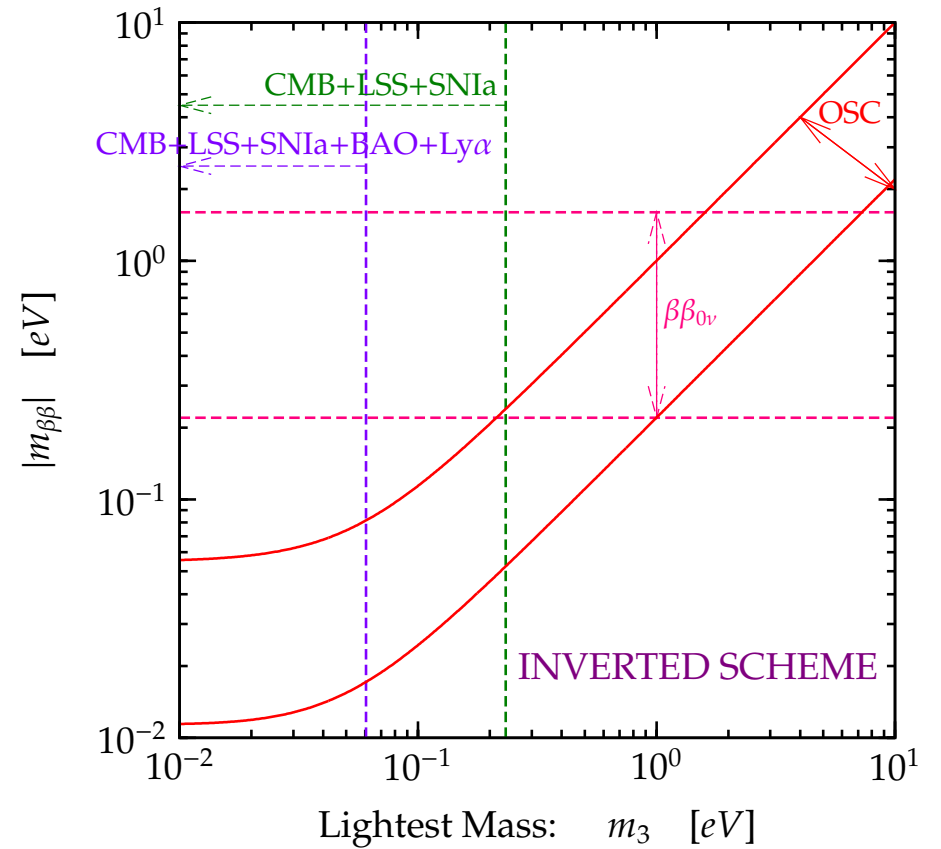
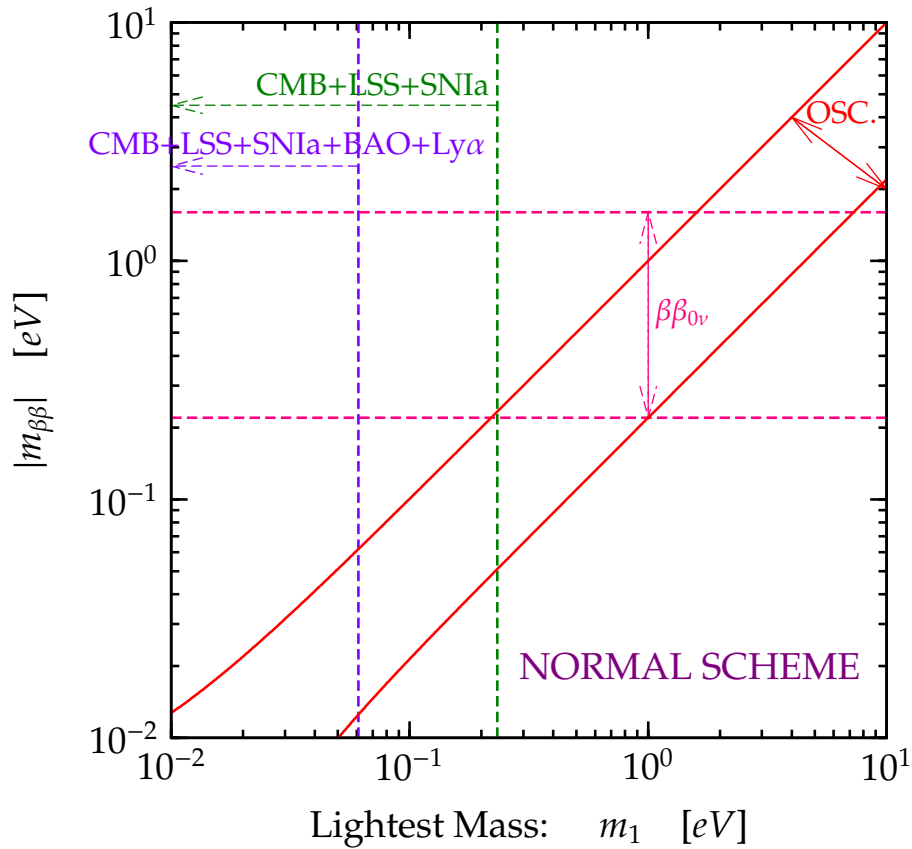
[PLB 586 (2004) 198]

the indication must be checked by other experiments

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12 \quad \implies \quad 0.22 \text{ eV} \lesssim |m_{\beta\beta}| \lesssim 1.6 \text{ eV}$$

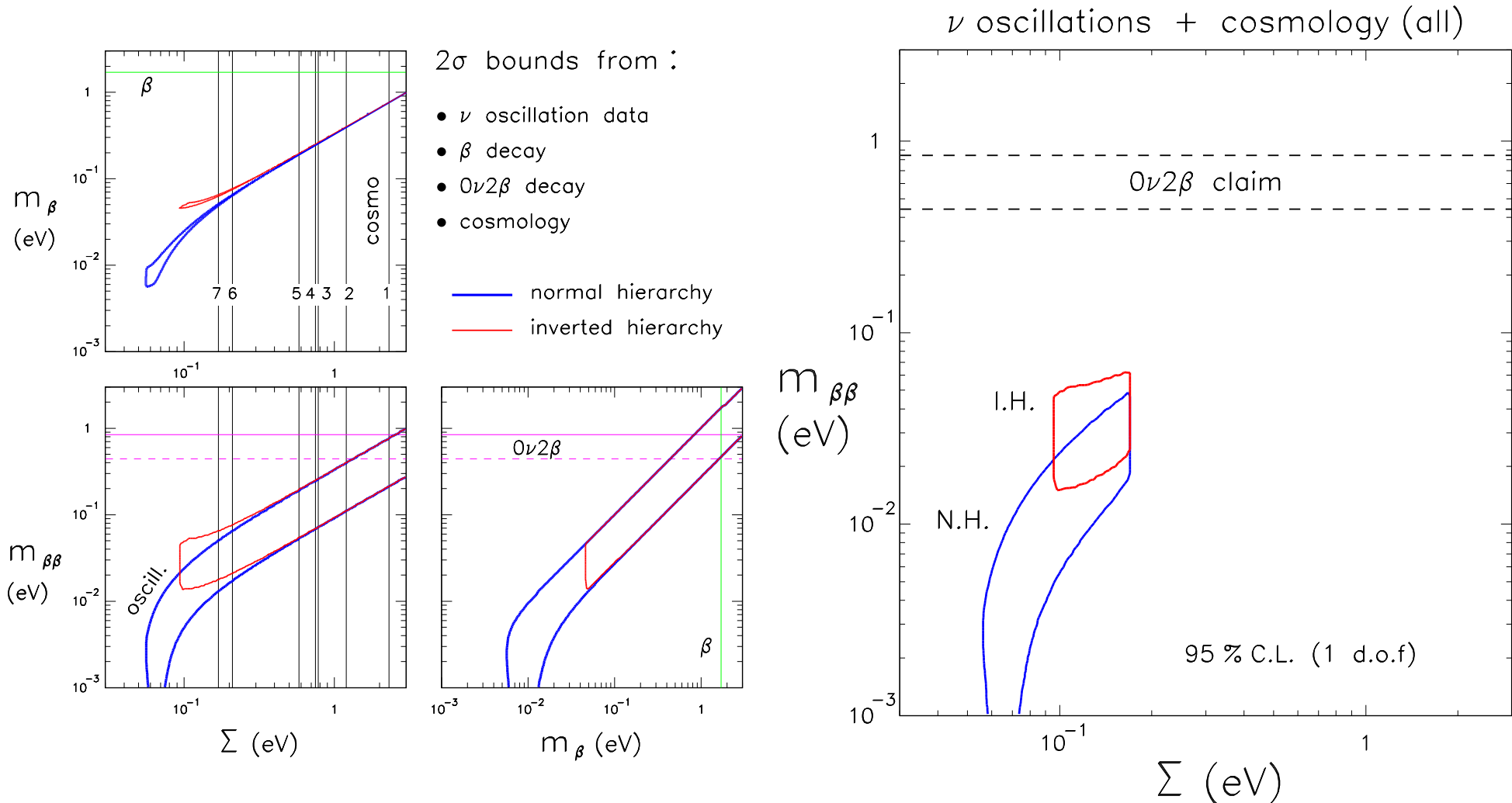
if confirmed, very exciting (Majorana ν and large mass scale)

Indication of $\beta\beta_{0\nu}$ Decay: $0.22 \text{ eV} \lesssim |m_{\beta\beta}| \lesssim 1.6 \text{ eV}$ ($\sim 3\sigma$ range)



tension among oscillation data, CMB+LSS+Ly α and $\beta\beta_{0\nu}$ signal

Case	Cosmological data set	Σ bound (2σ)
1	WMAP	< 2.3 eV
2	WMAP + SDSS	< 1.2 eV
3	WMAP + SDSS + SN _{Riess} + HST + BBN	< 0.78 eV
4	CMB + LSS + SN _{Astier}	< 0.75 eV
5	CMB + LSS + SN _{Astier} + BAO	< 0.58 eV
6	CMB + LSS + SN _{Astier} + Ly- α	< 0.21 eV
7	CMB + LSS + SN _{Astier} + BAO + Ly- α	< 0.17 eV

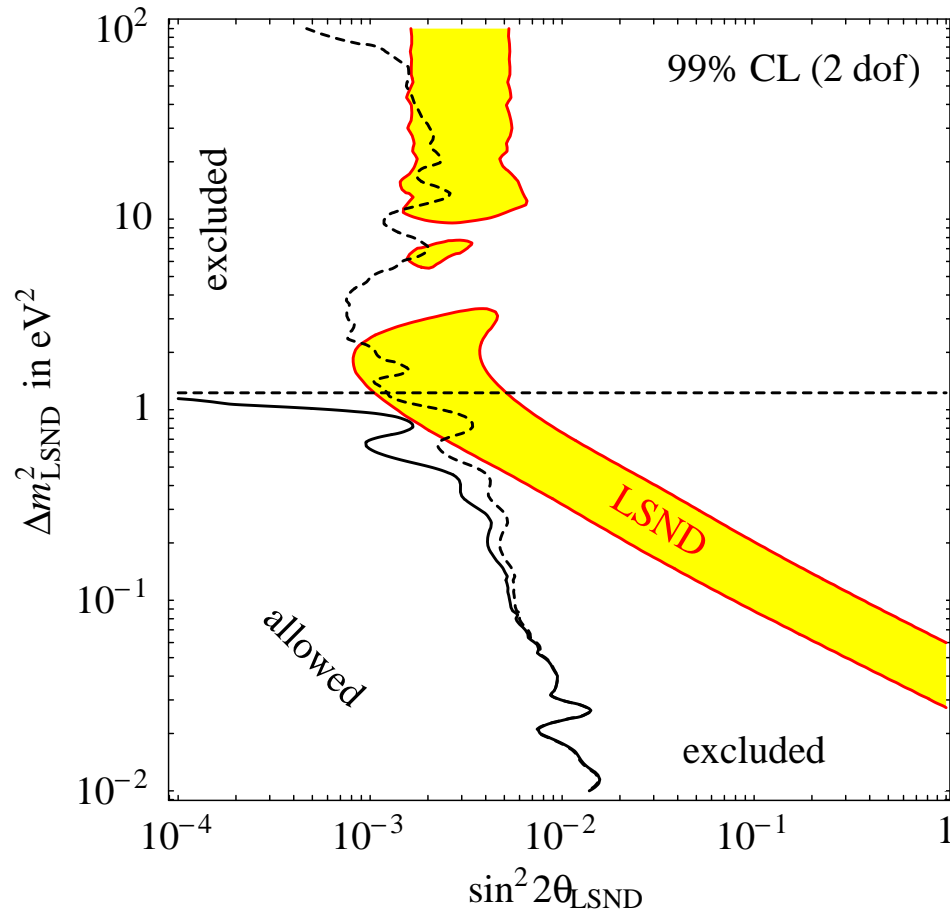


[Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, Slosar, hep-ph/0608060]

LSND

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

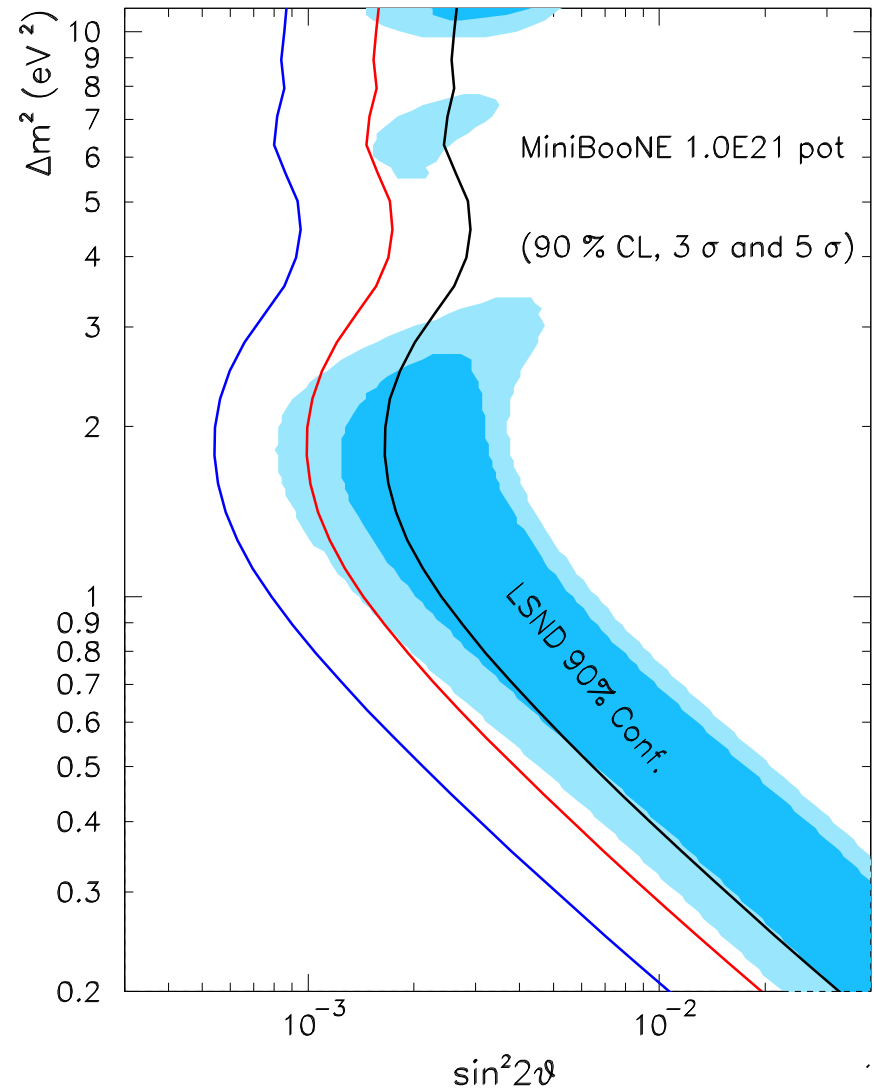
$$\Delta m_{\text{LSND}}^2 \gtrsim 0.1 \text{ eV}^2 (\gg \Delta m_{\text{ATM}}^2 \gg \Delta m_{\text{SUN}}^2)$$



[Strumia, Vissani, hep-ph/0606054]

3+1 scheme

bounds from ν experiments
and cosmology



MiniBooNE $\nu_\mu \rightarrow \nu_e$ [hep-ex/0406048]

Conclusions

$\nu_e \rightarrow \nu_\mu, \nu_\tau$ with $\Delta m_{\text{SUN}}^2 \simeq 8.3 \times 10^{-5} \text{ eV}^2$ (solar ν , KamLAND)

$\nu_\mu \rightarrow \nu_\tau$ with $\Delta m_{\text{ATM}}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$ (atm. ν , K2K, MINOS)



Bilarge 3ν -Mixing with $|U_{e3}|^2 \ll 1$ (CHOOZ)

β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay $\implies m_\nu \lesssim 1 \text{ eV}$

FUTURE

Theory: Why lepton mixing \neq quark mixing?

(Due to Majorana nature of ν 's?)

Why only $|U_{e3}|^2 \ll 1$?

Improve uncertainties in calculation of $\mathcal{M}_{0\nu}$!

Exp.: LSND?

Measure $|U_{e3}| > 0 \implies$ CP viol., matter effects, mass hierarchy

Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale

Improve β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay measurements