Theory and Phenomenology of Neutrino Oscillations and Masses

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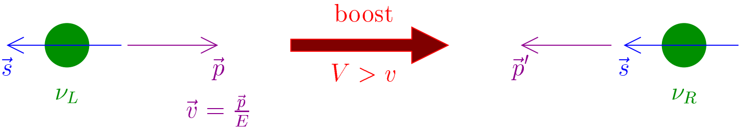
Outline

- Brief Introduction on Neutrino Masses
- Solar Neutrinos and KamLAND
- Atmospheric Neutrinos, K2K and MINOS
- Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Tritium Beta-Decay
- Neutrinoless Double-Beta Decay
- Cosmological Bound on Neutrino Masses
- LSND and MiniBooNE
- Conclusions

Standard Model: Massless Neutrinos



Standard Model: $\nu_L, \nu_L^c = (\nu^c)_R \quad \Longrightarrow \quad$ no Dirac mass term

$$\mathcal{L}^{\mathrm{D}} \sim m^{\mathrm{D}} \overline{\nu_L} \nu_R \qquad \text{(no } \nu_R,\ (\nu^c)_L\text{)}$$

Majorana Neutrino: $\nu \equiv \nu^c$

$$(\nu^c)_R \equiv \nu_R \quad \Longrightarrow \quad \text{Majorana mass term}$$

$$\mathcal{L}^{\mathrm{M}} \sim m^{\mathrm{M}} \overline{\nu_L} \nu_L^c = m^{\mathrm{M}} \overline{\nu_L} (\nu^c)_R$$

Standard Model: Majorana mass term not allowed by $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

(no Higgs triplet)

Extension of the SM: Massive Neutrinos

Standard Model can be extended with ν_R (e_L, e_R; u_L, u_R; d_L, d_R; ...)

$\nu_L + \nu_R \Rightarrow$ Dirac neutrino mass term $\mathcal{L}^{\mathrm{D}} \sim m^{\mathrm{D}} \overline{\nu_L} \nu_R \Rightarrow m^{\mathrm{D}} \lesssim 100\,\mathrm{GeV}$

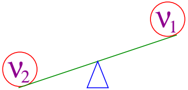
surprise: Majorana neutrino mass for ν_R is allowed! $\mathcal{L}^{\mathrm{M}}_R \sim m^{\mathrm{M}}_R \overline{(\nu^c)_L} \nu_R$

total neutrino mass term $\mathcal{L}^{\mathrm{D+M}} \sim \begin{pmatrix} \overline{\nu_L} & \overline{(\nu^c)_L} \end{pmatrix} \begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m^{\mathrm{M}}_R \end{pmatrix} \begin{pmatrix} (\nu^c)_R \\ \nu_R \end{pmatrix}$

m^{M}_R can be arbitrarily large (not protected by SM symmetries)

$m^{\mathrm{M}}_R \sim$ scale of new physics beyond Standard Model $\Rightarrow m^{\mathrm{M}}_R \gg m^{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m^{\mathrm{M}}_R \end{pmatrix} \quad \Rightarrow \quad m_1 \simeq \dfrac{(m^{\mathrm{D}})^2}{m^{\mathrm{M}}_R}, \quad m_2 \simeq m^{\mathrm{M}}_R$



see-saw mechanism

natural explanation of
smallness of neutrino masses

massive neutrinos are Majorana!

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

Lepton Numbers

Standard Model: Lepton numbers are conserved

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$((\nu^c)_e\,,\,e^+)$	-1	0	0
$((\nu^c)_\mu\,,\,\mu^+)$	0	-1	0
$((\nu^c)_\tau\,,\,\tau^+)$	0	0	-1

$$\boxed{L = L_e + L_\mu + L_\tau}$$

Dirac mass term $m^{\mathrm{D}}\overline{\nu_L}\nu_R \to \begin{pmatrix}\overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}}\end{pmatrix} \begin{pmatrix} m^{\mathrm{D}}_{ee} & m^{\mathrm{D}}_{e\mu} & m^{\mathrm{D}}_{e\tau} \\ m^{\mathrm{D}}_{\mu e} & m^{\mathrm{D}}_{\mu\mu} & m^{\mathrm{D}}_{\mu\tau} \\ m^{\mathrm{D}}_{\tau e} & m^{\mathrm{D}}_{\tau\mu} & m^{\mathrm{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix}\nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R}\end{pmatrix}$

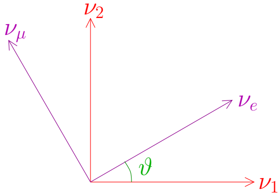
L_e, L_μ, L_τ are not conserved, but L is conserved $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Majorana mass term $m^{\mathrm{M}}\overline{\nu_L}(\nu^c)_R \to \begin{pmatrix}\overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}}\end{pmatrix} \begin{pmatrix} m^{\mathrm{M}}_{ee} & m^{\mathrm{M}}_{e\mu} & m^{\mathrm{M}}_{e\tau} \\ m^{\mathrm{M}}_{\mu e} & m^{\mathrm{M}}_{\mu\mu} & m^{\mathrm{M}}_{\mu\tau} \\ m^{\mathrm{M}}_{\tau e} & m^{\mathrm{M}}_{\tau\mu} & m^{\mathrm{M}}_{\tau\tau} \end{pmatrix} \begin{pmatrix}(\nu^c_e)_R \\ (\nu^c_\mu)_R \\ (\nu^c_\tau)_R\end{pmatrix}$

L, L_e, L_μ, L_τ are not conserved $L(\nu^c_\alpha) = -L(\nu_\beta) \Rightarrow |\Delta L| = 2$

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k}\,|\nu_k\rangle \qquad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

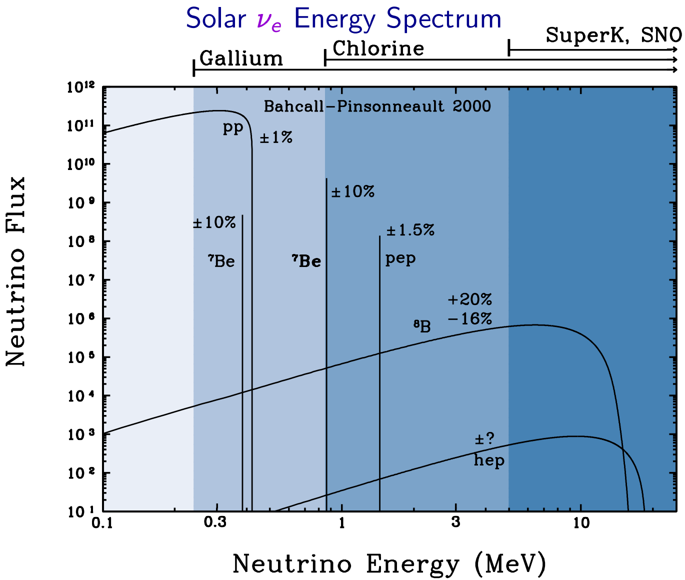
$$\boxed{\begin{aligned} |\nu_e\rangle &= \cos\vartheta\,|\nu_1\rangle + \sin\vartheta\,|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\vartheta\,|\nu_1\rangle + \cos\vartheta\,|\nu_2\rangle \end{aligned}}$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

Transition Probability: $$P_{\nu_e\to\nu_\mu} = P_{\nu_\mu\to\nu_e} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probabilities: $$P_{\nu_e\to\nu_e} = P_{\nu_\mu\to\nu_\mu} = 1 - P_{\nu_e\to\nu_\mu}$$

Solar Neutrinos



[J.N. Bahcall, http://www.sns.ias.edu/~jnb]

Homestake+Kam.+GALLEX+SAGE+Super-K+SNO

$$\Phi^{\rm SNO}_{\nu_e} = 1.76 \pm 0.11 \times 10^6\,{\rm cm}^{-2}\,{\rm s}^{-1}$$

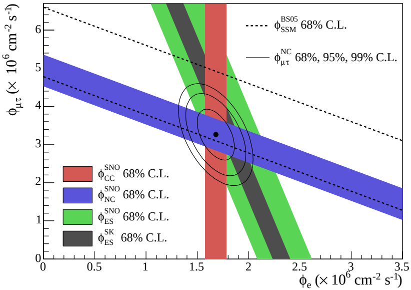
$$\Phi^{\rm SNO}_{\nu_e,\nu_\mu,\nu_\tau} = 5.09 \pm 0.66 \times 10^6\,{\rm cm}^{-2}\,{\rm s}^{-1}$$

SNO solved
solar neutrino problem

$\Downarrow$

Neutrino Physics
(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]



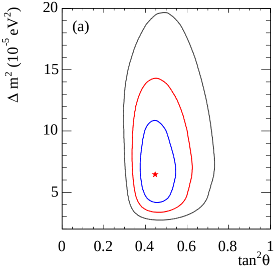
$\nu_e \to \nu_\mu, \nu_\tau$ oscillations

$\Downarrow$

LMA: Large Mixing Angle solution

$$\Delta m^2 \simeq 6.5^{+4.4}_{-2.3} \times 10^{-5}\,{\rm eV}^2$$

$$\tan^2\vartheta \simeq 0.45^{+0.09}_{-0.08}$$



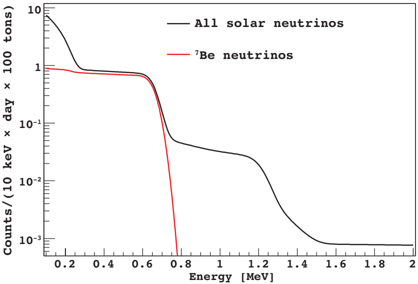
[SNO, PRC 72 (2005) 055502, nucl-ex/0502021]

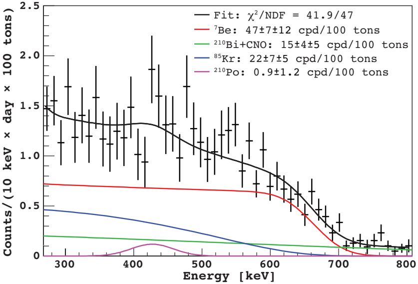
BOREXino

[BOREXino, arXiv:0708.2251]

Real-time measurement of ^{7}Be solar neutrinos (0.862 MeV) $E_{\text{th}} \simeq 0.25\,\text{MeV}$

$$\nu + e \to \nu + e \qquad E = 0.862\,\text{MeV} \quad \Longrightarrow \quad \sigma_{\nu_e} \simeq 5.5\,\sigma_{\nu_\mu,\nu_\tau}$$





$n_{\text{the}}^{\text{no-osc}} = 75 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1}$ $\qquad n_{\text{exp}} = 47 \pm 7 \pm 12\,\text{day}^{-1}\,(100\,\text{tons})^{-1}$

$n_{\text{the}}^{\text{osc}} = 49 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1}$ $\qquad (n_{\text{the}}^{\text{no-osc}} - n_{\text{exp}})/\Delta n \simeq 1.9$

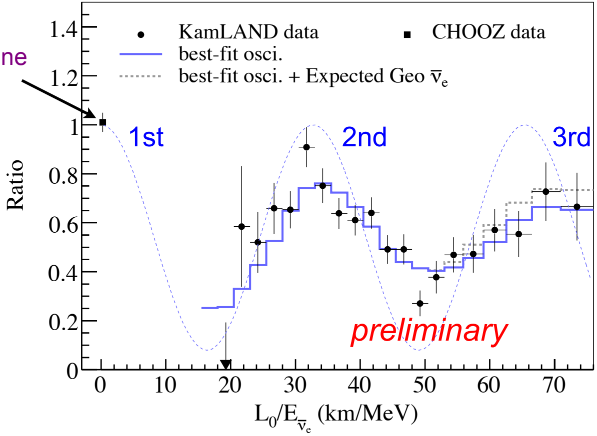
KamLAND

Reactor $\bar{\nu}_e \to \bar{\nu}_e$ confirmation of LMA (December 2002)

53 nuclear power reactors in Japan and Korea $\to$ Kamioka Mine

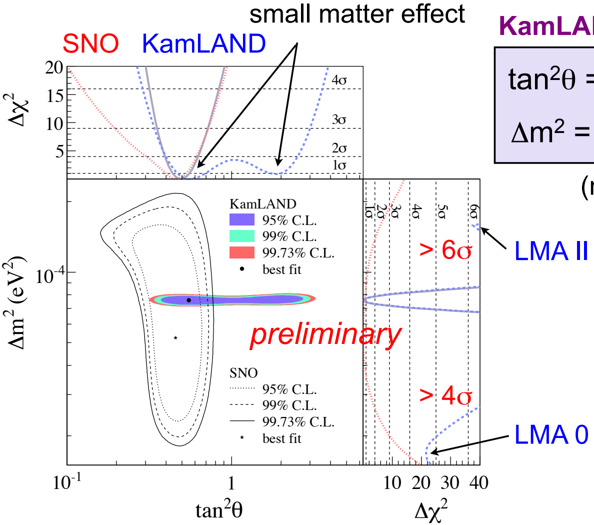
$\langle L \rangle \simeq 180\,\text{km}$ $\langle E \rangle \simeq 4\,\text{MeV}$

short baseline experiment



hypothetical single reactor at 180 km

[I. Shimizu (KamLAND), TAUP 2007]



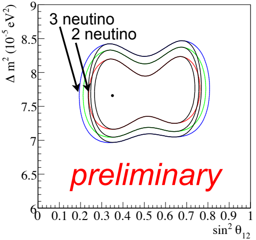
KamLAND only

$$\tan^2\theta = 0.56^{+0.14}_{-0.09}$$

$$\Delta m^2 = 7.58^{+0.21}_{-0.20} \times 10^{-5}\ \mathrm{eV}^2$$

(marginalized error)

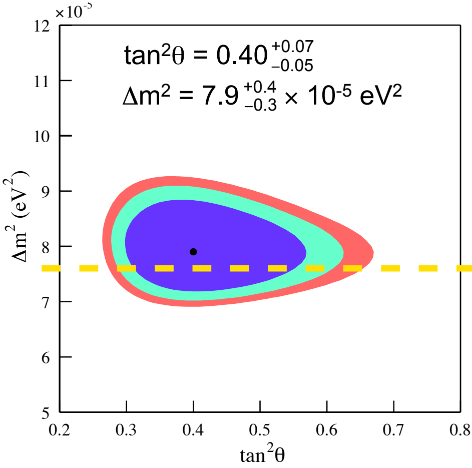
3 neutrino effect



same result for Δm^2

[I. Shimizu (KamLAND), TAUP 2007]

KamLAND 2004



This result

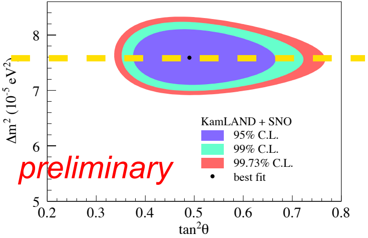
KamLAND + SNO

$$\tan^2\theta = 0.49^{+0.07}_{-0.05}$$

$$\Delta m^2 = 7.59^{+0.20}_{-0.21} \times 10^{-5}\ \text{eV}^2$$

Δm^2 : systematic uncertainty 2.0%

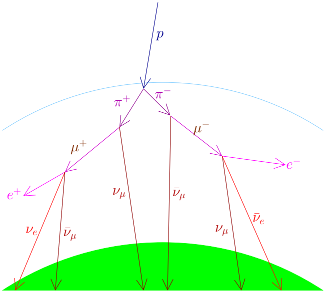
dominated by linear energy scale uncertainty

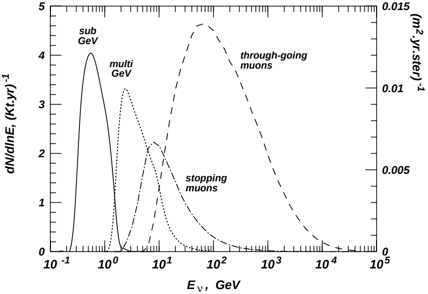


Δm^2 is measured at 2.8% precision by KamLAND

[I. Shimizu (KamLAND), TAUP 2007]

Atmospheric Neutrinos





$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1\,\text{GeV}$$

uncertainty on ratios: $\sim 5\%$

uncertainty on fluxes: $\sim 30\%$

ratio of ratios

$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

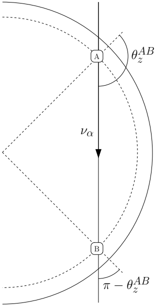
$$R^{\text{K}}_{\text{sub-GeV}} = 0.60 \pm 0.07 \pm 0.05$$

[Kamiokande, PLB 280 (1992) 146]

$$R^{\text{K}}_{\text{multi-GeV}} = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, PLB 335 (1994) 237]

Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1\,\text{GeV} \Rightarrow$ isotropic flux of cosmic rays

$$\phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\pi - \theta_z^{AB}) \qquad \phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\theta_z^{AB})$$

$$\Downarrow$$

$$\phi^{(A)}_{\nu_\alpha}(\theta_z) = \phi^{(A)}_{\nu_\alpha}(\pi - \theta_z)$$

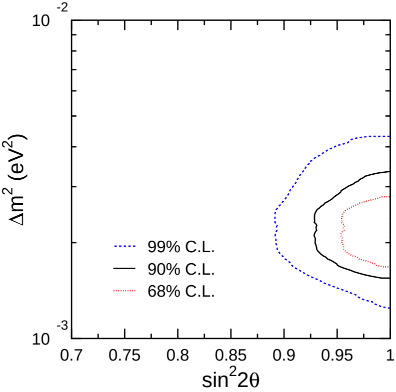
(December 1998)

$$\boxed{A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}}\right) = -0.296 \pm 0.048 \pm 0.01}$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

Fit of Super-Kamiokande Atmospheric Data



$\nu_\mu \to \nu_\tau$

Best Fit: $\begin{cases} \Delta m^2 = 2.1 \times 10^{-3}\,\text{eV}^2 \\ \sin^2 2\theta = 1.0 \end{cases}$

1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of ν_τ CC Int. is Difficult:

- $E_{\text{th}} = 3.5\,\text{GeV} \Longrightarrow \sim 20\text{events/yr}$
- τ-Decay $\Longrightarrow$ Many Final States

ν_τ-Enriched Sample

$$N_{\nu_\tau}^{\text{the}} = 78 \pm 26 \ @\ \Delta m^2 = 2.4 \times 10^{-3}\,\text{eV}^2$$

$$\boxed{N_{\nu_\tau}^{\text{exp}} = 138^{+50}_{-58}}$$

$$N_{\nu_\tau} > 0 \quad @ \quad 2.4\sigma$$

[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

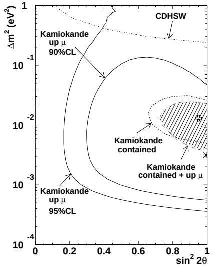
Future Check: OPERA ($\nu_\mu \to \nu_\tau$)

CERN to Gran Sasso (CNGS)

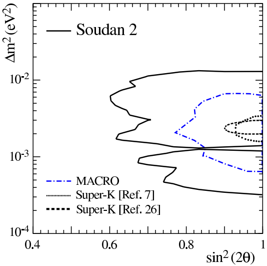
$L \simeq 732\,\text{km} \qquad \langle E \rangle \simeq 18\,\text{GeV}$

[NJP 8 (2006) 303, hep-ex/0611023]

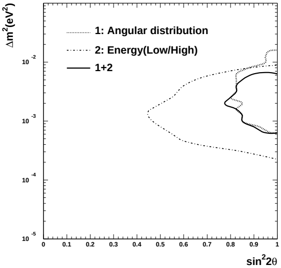
Kamiokande, Soudan-2, MACRO and MINOS



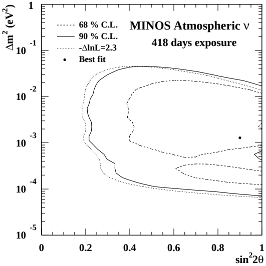
[Kamiokande, PRL 81 (1998) 2016, hep-ex/9806038]



[Soudan 2, PRD 72 (2005) 052005, hep-ex/0507068]



[MACRO, PLB 566 (2003) 35, hep-ex/0304037]



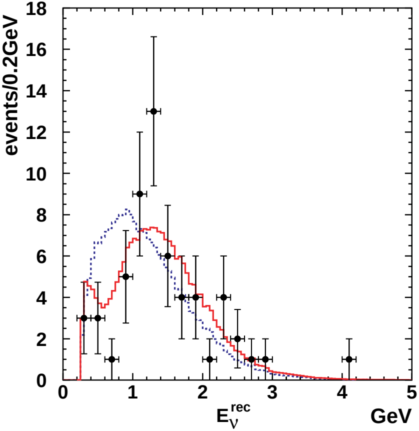
[MINOS, PRD 73 (2006) 072002, hep-ex/0512036]

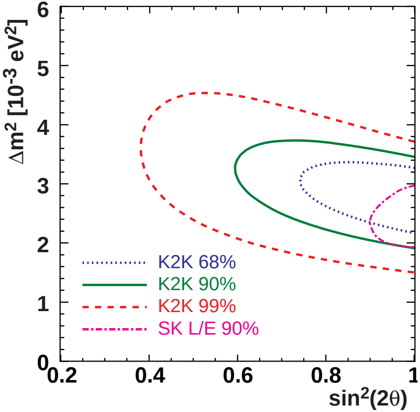
K2K

KEK to Kamioka (Super-Kamiokande) $\nu_\mu \to \nu_\mu$

$L \simeq 250\,\text{km} \qquad \langle E \rangle \simeq 1.3\,\text{GeV}$

confirmation of atmospheric allowed region (June 2002)





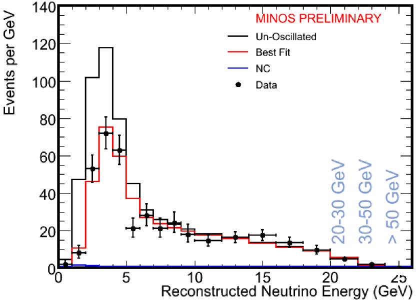
best fit: $\Delta m^2 = 2.75 \times 10^{-3}\,\text{eV} \quad \sin^2 2\vartheta = 1.00$

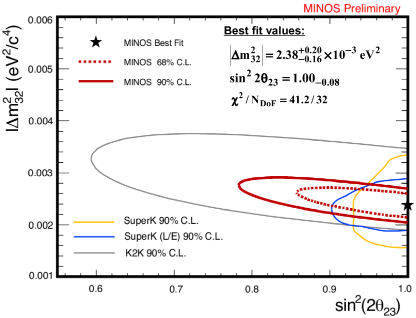
[K2K, PRD 74 (2006) 072003, hep-ex/0606032]

MINOS

Fermilab to Soudan Mine (Minnesota) $\nu_\mu \to \nu_\mu$

$L \simeq 730\,\text{km}$ $\langle E \rangle \simeq 3.5\,\text{GeV}$





[MINOS, TAUP 2007]

Experimental Evidences of Neutrino Oscillations

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

Reactor
$\bar{\nu}_e$ disappearance

$$\left.\begin{array}{c}\begin{pmatrix}\text{Homestake}\\ \text{Kamiokande/Super-K}\\ \text{GALLEX/GNO}\\ \text{SAGE}\\ \text{Super-Kamiokande}\\ \text{SNO}\\ \text{BOREXino}\end{pmatrix}\\ \\ \text{(KamLAND)}\end{array}\right\} \xrightarrow{2\sigma} \left\{\begin{array}{c}\Delta m^2_{\text{SUN}} = 7.59\,(1 \pm 0.03) \times 10^{-5}\,\text{eV}^2\\ \sin^2\vartheta_{\text{SUN}} = 0.49\,\left(1^{+0.14}_{-0.10}\right)\\ \text{[I. Shimizu (KamLAND), TAUP 2007]}\end{array}\right.$$

Atmospheric
$\nu_\mu \to \nu_\tau$

Accelerator
ν_μ disappearance

$$\left.\begin{array}{c}\begin{pmatrix}\text{Kamiokande}\\ \text{IMB}\\ \text{Super-Kamiokande}\\ \text{MACRO}\\ \text{Soudan-2}\end{pmatrix}\\ \\ \text{(K2K \& MINOS)}\end{array}\right\} \xrightarrow{2\sigma} \left\{\begin{array}{c}\Delta m^2_{\text{ATM}} = 2.6\,\left(1^{+0.14}_{-0.15}\right) \times 10^{-3}\,\text{eV}^2\\ \sin^2\vartheta_{\text{ATM}} = 0.45\,\left(1^{+0.35}_{-0.20}\right)\\ \text{[Fogli et al, PRD 75 (2007) 053001, hep-ph/0608060]}\end{array}\right.$$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

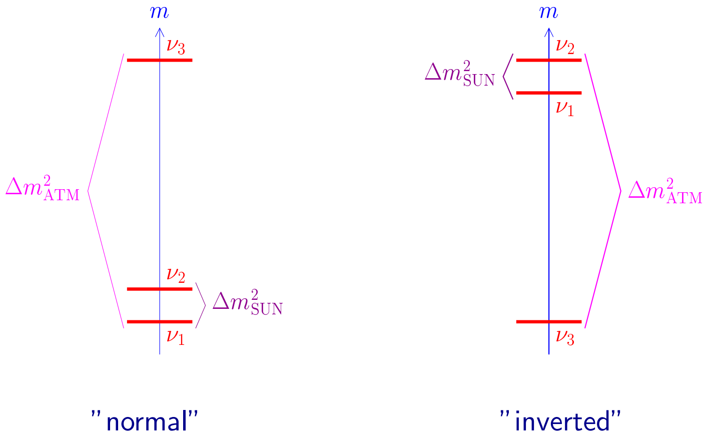
three flavor fields ν_e, ν_μ, ν_τ

three massive fields ν_1, ν_2, ν_3

$$\Delta m^2_{\text{SUN}} = \Delta m^2_{21} \simeq 7.6 \times 10^{-5}\,\text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

Allowed Three-Neutrino Schemes



different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

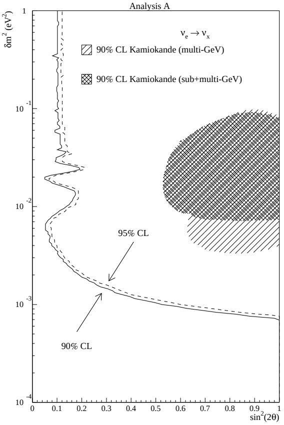
$$\begin{array}{rc} \text{SUN} \rightarrow & \\ & U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ & \uparrow \\ & \text{ATM} \end{array}$$

CHOOZ: $\begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 \lesssim 5 \times 10^{-2} \text{ for } \Delta m^2 \gtrsim 2 \times 10^{-2}\,\text{eV}^2}$$

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!



[CHOOZ, PLB 466 (1999) 415]

see also [Palo Verde, PRD 64 (2001) 112001]

TWO-NEUTRINO SOLAR and ATMOSPHERIC ν OSCILLATIONS ARE OK!

$$\sin^2\vartheta_{\text{SUN}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \simeq |U_{e2}|^2 \qquad \sin^2\vartheta_{\text{ATM}} = |U_{\mu 3}|^2$$

[Bilenky, C.G, PLB 444 (1998) 379]

[Guo, Xing, PRD 67 (2003) 053002]

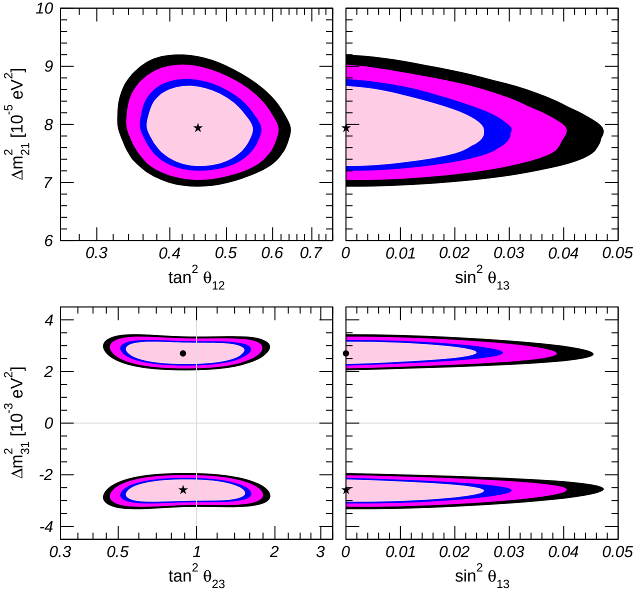
Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\mathrm{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} = \vartheta_{\mathrm{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\mathrm{SUN}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

Global Fit of Oscillation Data: Bilarge Mixing



[Gonzalez-Garcia, Maltoni, arXiv:0704.1800]

$$\Delta m^2_{21} = 7.9\,^{+0.27}_{-0.28}\ \left(^{+1.1}_{-0.89}\right) \times 10^{-5}\,\mathrm{eV}^2$$

$$\left|\Delta m^2_{31}\right| = 2.6 \pm 0.2\,(0.6) \times 10^{-3}\,\mathrm{eV}^2$$

$$\theta_{12} = 33.7 \pm 1.3\,\left(^{+4.3}_{-3.5}\right)$$

$$\theta_{23} = 43.3^{+4.3}_{-3.8}\ \left(^{+9.8}_{-8.8}\right)$$

$$\theta_{13} = 0^{+5.2}_{-0.0}\,\left(^{+11.5}_{-0.0}\right)$$

$$|U|_{90\%} = \begin{pmatrix} 0.81 - 0.85 & 0.53 - 0.58 & 0.00 - 0.12 \\ 0.32 - 0.49 & 0.52 - 0.69 & 0.60 - 0.76 \\ 0.27 - 0.46 & 0.47 - 0.64 & 0.65 - 0.80 \end{pmatrix}$$

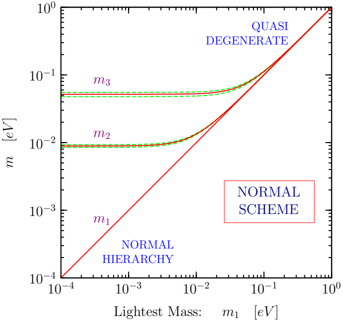
$$|U|_{3\sigma} = \begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0.00 - 0.20 \\ 0.25 - 0.53 & 0.47 - 0.73 & 0.56 - 0.79 \\ 0.21 - 0.51 & 0.42 - 0.69 & 0.61 - 0.83 \end{pmatrix}$$

[Gonzalez-Garcia, Maltoni, arXiv:0704.1800]

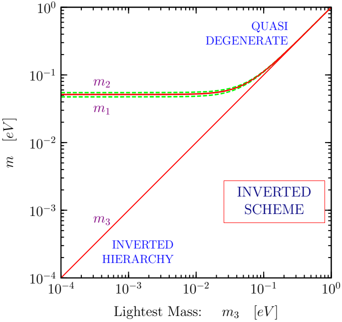
future: measure $\vartheta_{13} \neq 0 \Longrightarrow$ CP violation, matter effects, mass hierarchy

Absolute Scale of Neutrino Masses

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SUN}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

Tritium Beta-Decay

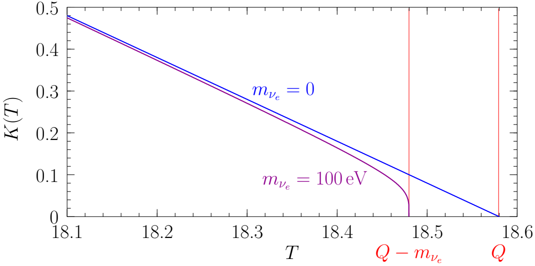
$^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58\,\text{keV} \qquad m_{\nu_e}^2 = m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$\boxed{m_{\nu_e} < 2.2\,\text{eV} \quad (95\%\ \text{C.L.})}$

Mainz & Troitsk

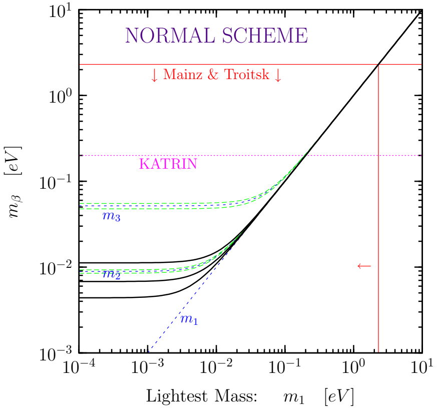
[Weinheimer, hep-ex/0210050]

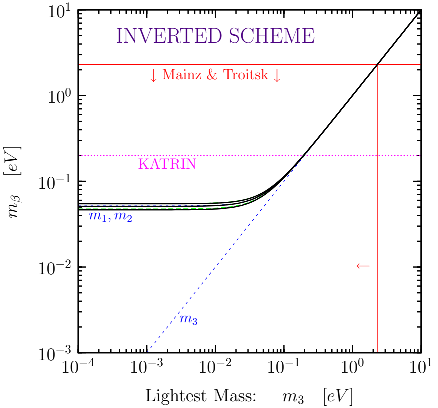
future: KATRIN (start 2010)

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_{\nu_e} \simeq 0.2\,\text{eV}$

$$m_{\nu_e}^2 = m_\beta^2 = |U_{e1}|^2 \, m_1^2 + |U_{e2}|^2 \, m_2^2 + |U_{e3}|^2 \, m_3^2$$

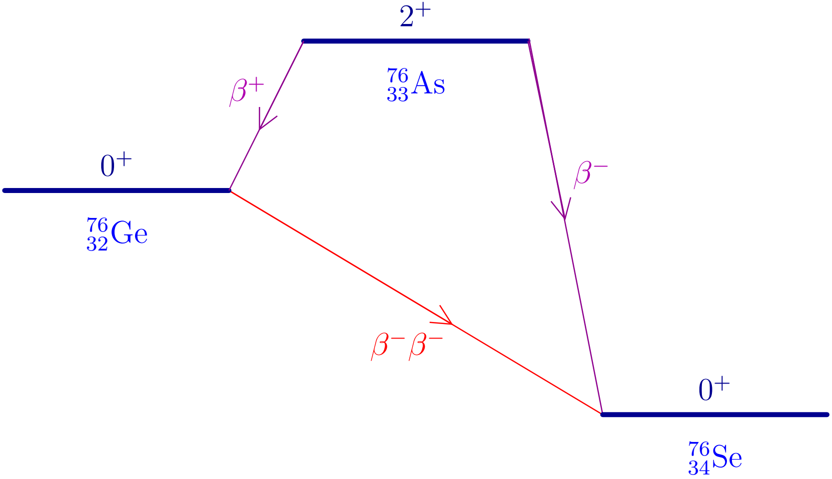




Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}$ eV $\implies$ NORMAL HIERARCHY

Neutrinoless Double-Beta Decay

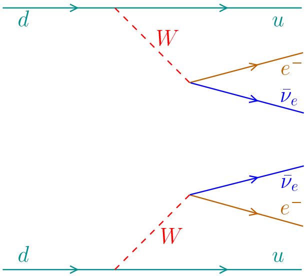


Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu} \, |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model



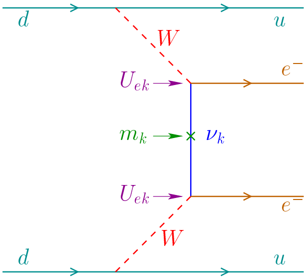
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

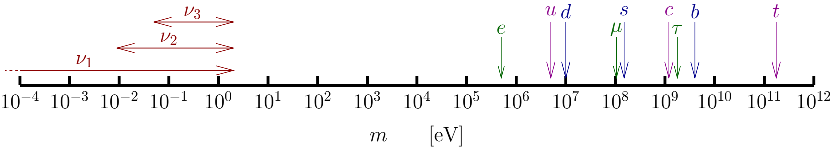
$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu} \, |\mathcal{M}_{0\nu}|^2 \, |m_{\beta\beta}|^2$$

effective
Majorana
mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$



Majorana Neutrino Mass?



known natural explanation of smallness of ν masses

New High Energy Scale $\mathcal{M}$ $\Rightarrow$ $\begin{cases} \text{See-Saw Mechanism (if } \nu_R\text{'s exist)} \\ \text{5-D Non-Renormaliz. Eff. Operator} \end{cases}$

both imply $\begin{cases} \text{Majorana } \nu \text{ masses} \iff |\Delta L| = 2 \iff \beta\beta_{0\nu} \text{ decay} \\ \text{see-saw type relation } m_\nu \sim \dfrac{\mathcal{M}_{\text{EW}}^2}{\mathcal{M}} \end{cases}$

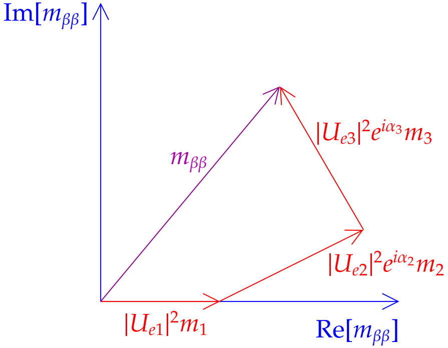
Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

Effective Majorana Neutrino Mass

$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$ complex U_{ek} $\Rightarrow$ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$



Experimental Bounds

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$\boxed{T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$\boxed{T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}}$$

CUORICINO (^{130}Te) [PRL 95 (2005) 142501]

$$\boxed{T^{0\nu}_{1/2} > 1.8 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.2 - 1.1\,\text{eV}}$$

NEMO 3 (^{100}Mo) [arXiv:0710.5604]

$$\boxed{T^{0\nu}_{1/2} > 5.8 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.6 - 0.9\,\text{eV}}$$

FUTURE EXPERIMENTS

CAMEO, CANDLES, COBRA, EXO, Majorana, XMASS

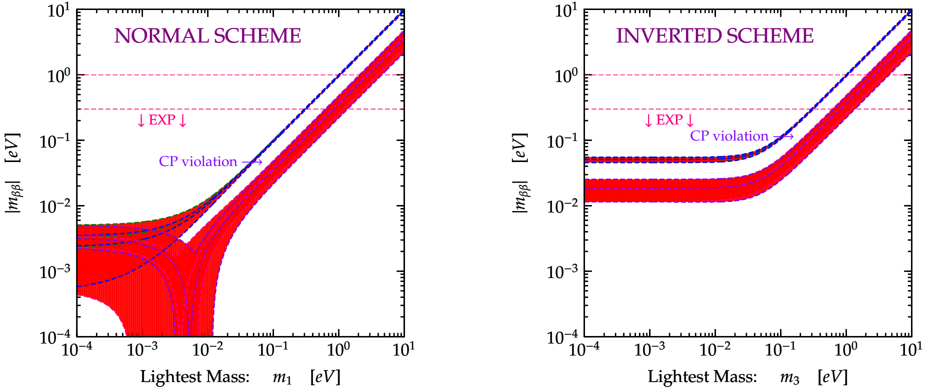
$|m_{\beta\beta}| \sim \text{few}\,10^{-1}\,\text{eV}$

CUORE, MOON, GEM, GERDA, SNO++, Super-NEMO

$|m_{\beta\beta}| \sim \text{few}\,10^{-2}\,\text{eV}$

Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

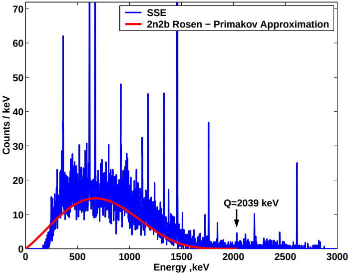


FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\mathrm{eV} \Longrightarrow$ NORMAL HIERARCHY

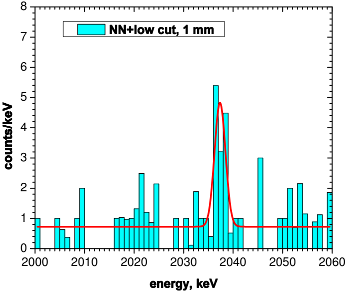
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409]

$T^{0\nu}_{1/2} = (2.23^{+0.44}_{-0.31}) \times 10^{25}$ y 6.5σ evidence [MPLA 21 (2006) 1547]



[PLB 586 (2004) 198]



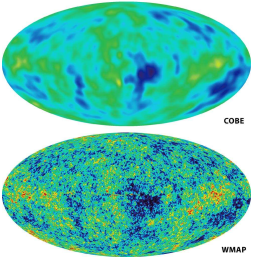
[MPLA 21 (2006) 1547]

the indication must be checked by other experiments

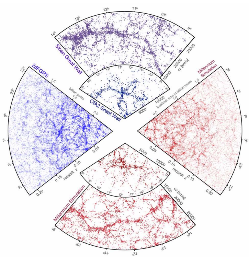
$|m_{\beta\beta}| = 0.32 \pm 0.03\,\text{eV}$ [MPLA 21 (2006) 1547]

if confirmed, very exciting (Majorana ν and large mass scale)

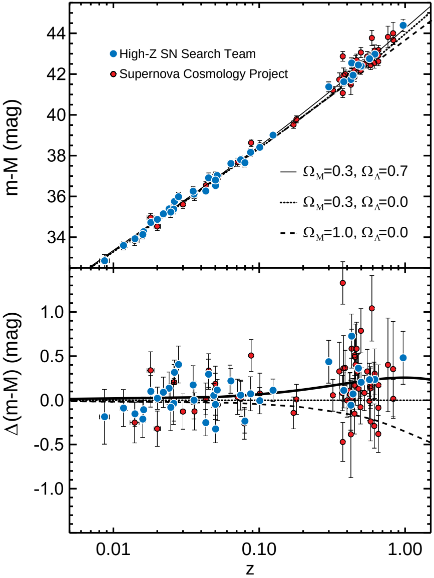
Cosmological Bound on Neutrino Masses



[WMAP, `http://map.gsfc.nasa.gov`]



[Springel, Frenk, White, Nature 440 (2006) 1137]



[`http://cfa-www.harvard.edu/supernova/`]

Relic Neutrinos

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$$

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \overset{\text{weak interactions freeze out}}{\sim} T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \Longrightarrow \underset{\text{neutrino decoupling}}{T_{\text{dec}} \sim 1\,\text{MeV}}$$

$$\text{Relic Neutrinos:} \quad T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\, T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$$

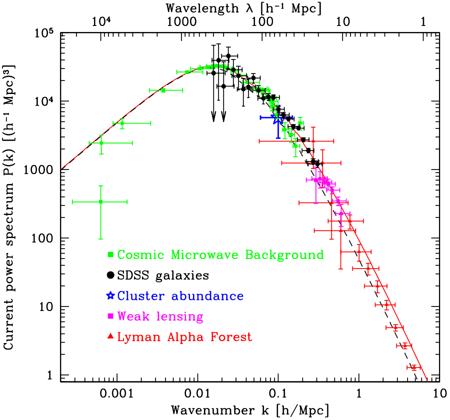
$$\text{number density:} \quad n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\, T_\nu^3 \simeq 112\,\text{cm}^{-3}$$

$$\underset{\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)}{\text{density contribution:}} \quad \Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\, m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \qquad \Longrightarrow \qquad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_{\mathrm{M}}^0 = 0.28\,,\ h = 0.72\,,\ \Omega_{\mathrm{B}}^0/\Omega_{\mathrm{M}}^0 = 0.16)$

Dashed Curve: $\sum_{k=1}^{3} m_k = 1\,\mathrm{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8 \left(\frac{\sum_k m_k}{1\,\mathrm{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for

$$k \gtrsim k_{\mathrm{nr}} \approx 0.026 \sqrt{\frac{m_\nu}{1\,\mathrm{eV}}}\sqrt{\Omega_m}\, h\,\mathrm{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

H_0 (Hubble Space Telescope Key Project)

CMBR (COBE, WMAP, Boomerang, DASI, MAXIMA, VSA, CBI, ACBAR)

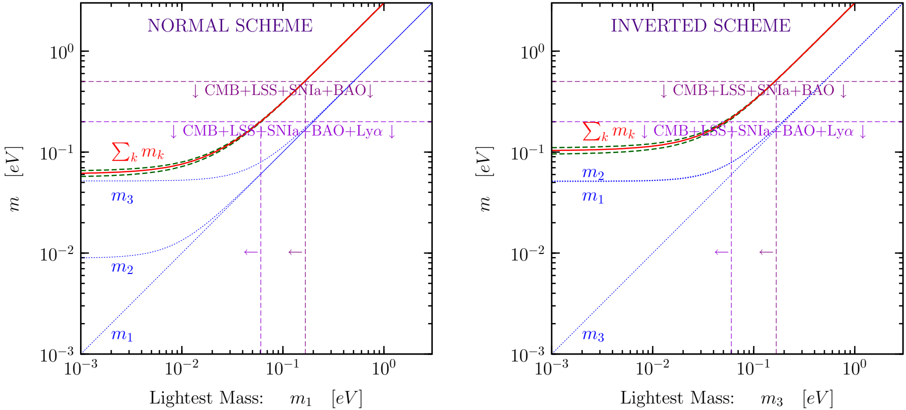
LSS (2dFGRS, SDSS)

SNIa (High-z SN Search Team, Supernova Cosmology Project)

$$\Lambda\text{CDM} \quad \Longrightarrow \quad \boxed{\sum_{k=1}^{3} m_k \lesssim 0.2 - 0.5\,\text{eV}}$$

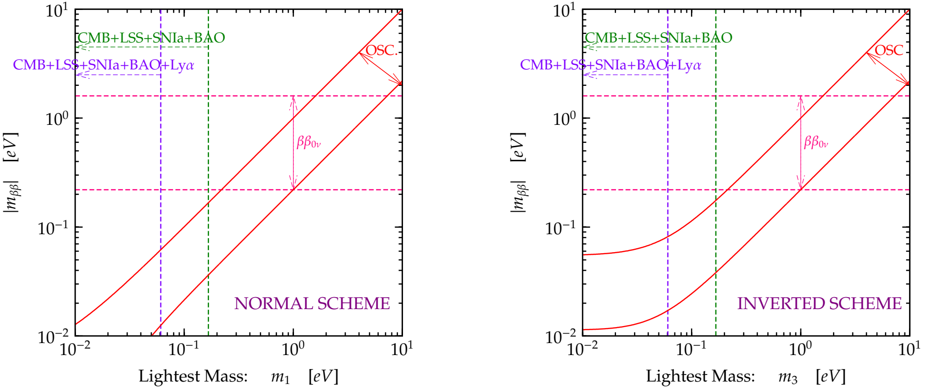
$$\sum_{k=1}^{3} m_k \lesssim 0.5\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+SNIa+BAO}$$

$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+SNIa+BAO+Ly}\alpha$$



FUTURE: IF $\sum_{k=1}^{3} m_k \lesssim 9 \times 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

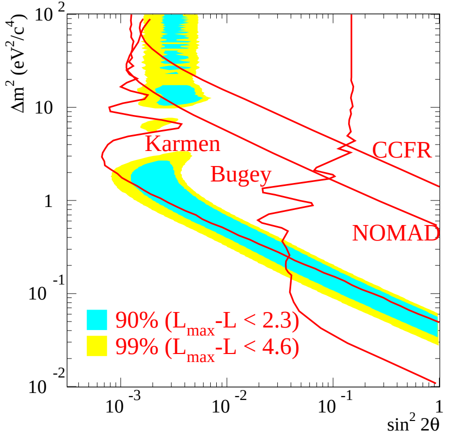
Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\mathrm{eV}$ $\quad (\sim 3\sigma$ range$)$



tension among oscillation data, CMB+LSS+BAO(+Lyα) and $\beta\beta_{0\nu}$ signal

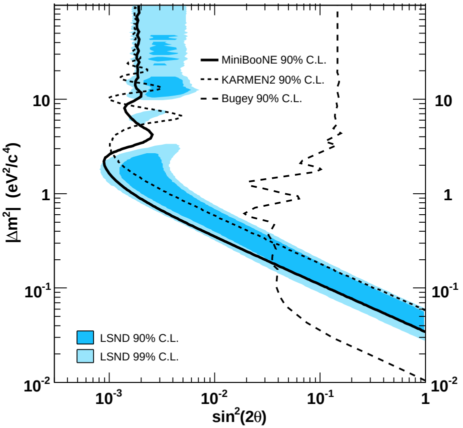
LSND and MiniBooNE

$\bar{\nu}_\mu \to \bar{\nu}_e$



[LSND, PRD 64 (2001) 112007, hep-ex/0104049]

$\nu_\mu \to \nu_e$



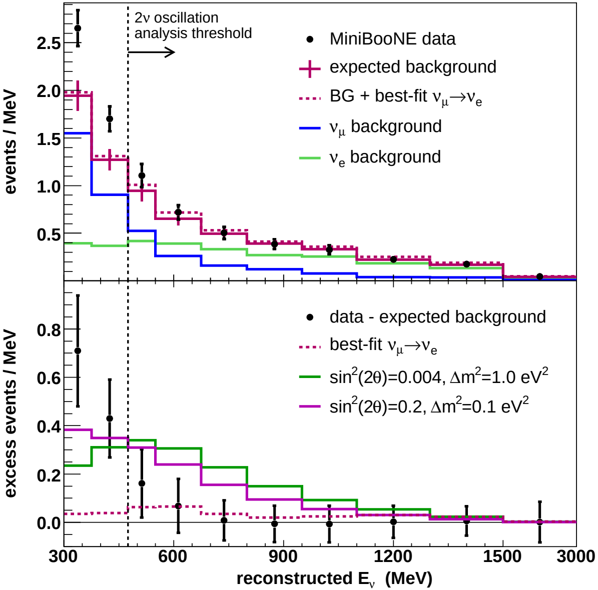
[MiniBooNE, PRL 98 (2007) 231801, arXiv:0704.1500]

$$\Delta m^2_{\text{LSND}} \gtrsim 0.1\,\text{eV}^2$$

$$\Delta m^2_{\text{LSND}} \gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SUN}}$$

$\bar{\nu}_\mu \to \bar{\nu}_e$ is running

MiniBooNE Low-Energy Anomaly



[MiniBooNE, PRL 98 (2007) 231801, arXiv:0704.1500]

Conclusions

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SUN}} \simeq 7.6 \times 10^{-5}\,\text{eV}^2$ (solar ν, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.5 \times 10^{-3}\,\text{eV}^2$ (atm. ν, K2K, MINOS)

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$ (CHOOZ)

β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay $\Longrightarrow m_\nu \lesssim 1\,\text{eV}$

FUTURE

Theory: Why lepton mixing $\neq$ quark mixing?
Why only $|U_{e3}|^2 \ll 1$?
Improve calculation of $\mathcal{M}_{0\nu}$!
LSND and MiniBooNE Low-Energy Anomaly?

Exp.: Measure $|U_{e3}| > 0 \Rightarrow$ CP violation, matter eff., mass hierarchy
Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale
Improve β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay measurements
LSND and MiniBooNE Low-Energy Anomaly?